COINCIDENT AND LEADING INDICATORS OF THE BARBADIAN BUSINESS CYCLE

by

GLADYS COTRIE
University of Guadeloupe

Presented at the 26th Annual Review Seminar
Research Department
Central Bank of Barbados
July 26-29, 2005

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Abstract

This paper constructs coincident and leading indicator indices for the Barbadian business cycle using the Stock and Watson (1989) method and a variant thereof. The results indicate that both procedures seem to provide indices that reflect the reference business cycle fairly well.

Key words: State Space model, Kalman Filter, Business Cycle, Coincident and Leading Indicators

1 Summer intern in the Research Department of the Central Bank of Barbados and a Master student in Economic Engineering of the Environment and Development at the University “Des Antilles et de la Guyane campus of Guadeloupe”. I would like to thank my supervisor Roland Craigwell, Ryan Straughn and Alain Maurin for guidance.
1. Introduction

An understanding of macroeconomic fluctuations provides an input into forecasting growth and recession phases of an economy, which are necessary ingredients for the formulation of macroeconomic policies. Important tools in this analysis are leading, coincident and lagging economic indicators. These indices have been used in industrialized countries, such as the U.S. and Canada, to understand and forecast the business cycle (see Gaudreault, Lamy and Liu, 2003; Stock and Watson, 1989). For most developing countries, especially the Caribbean islands like Barbados, these indices have not yet been developed. In this paper, an attempt is made to fill this void by establishing coincident and leading indicator indices for Barbados. The approach employed is based on econometric techniques and is due to Stock and Watson (1989), hereafter SW.

The structure of the paper is as follows. Section 2 is a brief review about coincident and leading indicators. Section 3 deals with the economic performance of Barbados. Section 4 presents a chronology of the Barbadian real GDP series. Section 5 discusses the SW method. Section 6 constructs coincident and leading indicators with the SW methodology. Section 7 develops a simplified version of SW approach, and compares these results with those from the SW estimation and section 8 concludes.

2. A Brief Review of the Literature on Coincident and Leading Indicators

The approach to developing cyclical indices originated with the U.S. National Bureau of Economic Research (NBER), pioneered by the work of Burns and Mitchell (1946) in the 1920s and 1930s and extended by economists, notably Moore and Skriskin (1967), in the 1950s and 1960s. These researchers combined a number of economic time series, selected on the basis of various criteria - economic significance, cyclical behaviour, data quality, timeliness and

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A coincident indicator is a economic index that generally has the same trend as the business cycle, such as industrial production. Leading indicators are industrial and economic statistics that are considered to rise or fall before the changes in economic growth rates and total business activity. They generally predict the future performance of economy activity six month into the future. A lagging indicator is a statistical measure of a country’s activity, which reflects the economic change with a delay of a defined period. For more details see the following website: http://www.aquanto.com/glossary/l.html.
availability - into coincident, leading and lagging economic indices using specific weighted schemes.

In the 1970s and 1980s, this approach spread to Europe and the Organisation for Economic Cooperation and Development (OECD) applied a modified version of the NBER method\(^\text{3}\) to its member countries. However, the selection and weighting processes of the NBER cyclical indicator procedure remained unchanged until Stock and Watson (1989) developed a new system of composite indices of coincident and leading indicators, as well as a recession index for the United States, using modern econometric techniques. The composite coincident economic index is based on econometric models that depict the state of the economy as an unobserved variable, which is common to several macroeconomic variables. Then, instead of utilising a weighted average of leading indicators as done in the NBER approach, the leading indicator index is derived from a vector autoregression (VAR) forecast of the change in the composite coincident index on past changes in the composite coincident economic index and other variables that have historically led the business cycle. If the coincident index truly reflects the state of the economy, then a good forecast of this coincident index should provide for a good leading index. This method, unlike the traditional approach, does not lack theoretical rigour, that is, it pays attention to economic theory in determining the relationship between the indicators and economic activity, as well as it does not rely heavily on subjective analysis, rather an econometric (scientific) approach is used (see Koopmans, 1947; Averbach, 1982; Leeuw, 1991 for further details).

The literature for developing countries is fairly scant, possibly because (i) the business cycle in developing countries are likely to be more dependent on weather patterns than cyclical fluctuations, as a result of the preponderance of agriculture in the production process (Mall, 1999); (ii) there are heavy restrictions on the quality and frequency of the data and; (iii) of difficulties in discerning any type of cycle or economic regularity because of the sudden crises and market gyrations typical of developing countries (Agenor, McDermott and Prasad, 2000).

However, increasingly, examples for developing countries are appearing. Mall (1999) and Dua and Banerji (1999, 2001) developed coincident and leading indicators for India, Simone (2000) for Argentina and more recently, Mongardini and Saadi-Sedik (2003) for Jordan. All of these studies have performed relatively well but their methodologies, unlike in the developed world, have not been officially adopted or tested to predict future business cycles. Since the year 1990, the Conference Board (and before that the Bureau of Economic Analysis) has regularly published leading, lagging and coincident index for the U.S.\(^\text{4}\)

3. The Economic Performance of Barbados

Despite its small size of 431 square kilometres, a population of less than 270,000 inhabitants and a meagre endowment of natural resources, Barbados’ development experience has been a true success story. It has diversified from a monoculture based on the production and export of raw sugar, to an economy driven by tourism and financial services. Barbados remains among the most developed countries in Latin America and the Caribbean, with levels of health, education, communication and social services comparable to those of industrialised countries. In fact, in 2004, Barbados was ranked 29\(^\text{th}\) among 177 countries in the United Nations Development Programme’s Human Development Index.

Graph 1: The Real GDP of Barbados

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The September 11 period 2000 to 2001. A world recession and the September 11 terrorist attacks put a damper on real value added of major sectors like tourism and manufacturing. Government had to increase expenditure to keep its main engines of growth going.

The post September 11 period. With government counter cyclical spending, tourism recovered and real output started to grow.

4. A Chronology of the Real GDP Series

Business cycles are recurrent sequences of alternating phases of expansions and contractions in the level of a time series, usually explained by the co-movements of other economic variables. This definition is based on the early work of Burns and Mitchell (1946) who wrote that:

"Business Cycles are a type of fluctuation found in the aggregate activity of nations that organize their work mainly in business enterprises: a cycle consists of expansions occurring at about the same time in many activities, followed by similarly general recessions, contractions and revivals which merge into the expansion phase of the next cycle; this sequence of changes is recurrent but not periodic; in duration business cycles vary from more than one year to ten or twelve years; they are not divisible into shorter cycles of similar character with amplitudes approximating their own" (pp.3).

Business cycle fluctuations are often measured using the real gross domestic product of an economy, on the assumption that it represents the economic strength of a nation. In this section a chronology of the Barbadian business cycle is developed using quarterly real GDP, measured in millions of Barbadian dollars and covering the period 1974 to 2003, with base year = 1974. The data source is the Central Bank of Barbados and all estimations are done utilising the RATS and EVIEWS statistical software packages. This analysis is basically an update of Craigwell and Maurin (2002) and therefore, it will necessarily be brief.

From Graph 1 above, it appears that there is a growing trend, many seasonal cycles and three possible business cycles (namely, 1974 –1982, 1983-1992 and 1993-2003). However, because of the many perturbations in the series, the cycles are not well defined, and the latter should be separated from the trend and the irregular components. Several filters of trend-cycle
decomposition are available but the Hodrick-Prescott (1980) filter is the one chosen here. This method consists of minimizing the variance of the time series around the trend. The minimization schedule is as follow:

\[
M in \left\{ \sum_{i=1}^{T} (y_i - y_i^*)^2 + \lambda \sum_{i=1}^{T-1} \left[ \left( y_{i+1}^* - y_i^* \right) - \left( y_{i+1} - y_i \right) \right]^2 \right\}
\]

where \( y_i \) is real GDP, \( y_i^* \) is the permanent component of \( y_i \) and \( \lambda \) is the lagrange multiplier, which divides the total fluctuations into long-term and short-term movements, with its value determined by the observed fluctuations. Hodrick and Prescod (1980) established a value of \( \lambda = 1600 \) for quarterly U.S. data, and this same number and other values were tried in Craigwell and Maurin (2002,2004,2005a, b) with no appreciable difference in the underlying results. As a result, this paper utilizes a value of 1600 for \( \lambda \).

Graph 2: GDP decomposition using the Hodrick-Prescott Filter

Graph 2 depicts the trend-cycle GDP decomposition and shows the separate components. The trend can be interpreted as potential output and the cycle reveals impulse response of shocks on the economy and all the seasonal cycles. After identifying the cycle, the next step is to discern the cycle by determining its peaks and troughs. Formally, a turning point occurs in a series when the deviation from trend reached a local maximum (peak) or a local minimum (trough). Table 1 gives these expansion and recession phases for real GDP.

**Table 1: Expansions and recessions of Barbados**

<table>
<thead>
<tr>
<th>Period</th>
<th>Phase of cycle</th>
<th>Year of peak and trough</th>
<th>Quarter time</th>
</tr>
</thead>
<tbody>
<tr>
<td>1992:4-2003:4</td>
<td>Expansion</td>
<td></td>
<td>44</td>
</tr>
</tbody>
</table>

Given that the cycle is defined by the distance of two troughs, then, the economy of Barbados shows two business cycles over the study’s sample period, that is, 1974:1 – 1982:4 (approximately 9 years) and 1982:4 – 1992:3 (approximately 10 years), confirming the existence of business cycles in the economic fluctuations of Barbados (see also Craigwell and Maurin, 2002; 2004;2005a, b). Indeed, they correspond to the stylised facts summarised in Section 3 above. The period 1974-1982 is an era of rapid growth and employment emanating from the diversification of sugarcane cultivation to manufacturing, tourism and financial services. 1983-1992 relates to a decline in real output because of the international recession that was associated with the stagnation of markets, the oil shocks and world-wide inflation. The period 1993 - 2003 is an era of expansion, driven mainly by the tourist industry and financial services, helped by the IMF austerity measures of the structural adjustment programme. Given these business cycles, indices of economic indicators can be built to forecast real economic activity.
5. The Stock and Watson Theoretical Approach to Economic Indicators

Many approaches have been used to compute a business cycle index using the Burns and Mitchell (1946)'s definition of the business cycle. The first and the most utilised is that provided by the NBER. Indeed, in 1937, Mitchell and Burns (1946) developed a list of 487 indicators that led, lagged or were coincident with the business cycle. The project embraced the concept that there is a business cycle or reference cycle that cannot be observed directly but can be measured by the consistent movement of many economic variables as the phases of growth change.

In the 1950s and 1960s, researchers from the NBER extended the concept by constructing indices from these indicators, weighting and adding together variables that consistently led, lagged or kept pace with the business cycle. This method estimates the index as a weighted average of individual indicators. Mathematically,

\[
X = \sum_{i=1}^{n} w_i I_i,
\]

where \( X \) is the composite index, \( I_i \) is the \( i^{th} \) indicator index and \( w_i \) is the weight allocated to \( I_i \).

Due to the atheoretical and unscientific nature of the above procedure, a second approach was initiated by Stock and Watson (1989, 1992). The composite coincident economic index (CEI) is based on an econometric model in which the “state of the economy” is an unobserved variable, which is common to several macroeconomic variables. The model relies on the fact that the fluctuations in these variables share a common element, which can be estimated. If the coincident index truly reflects the state of the economy, then a good forecast of this coincident index should make a good leading index. The co-movements at all leads and lags among the coincident variables are modelled as arising from a single common source \( C_t \), a scalar unobserved time series that can be thought of as the overall state of the economy. The structure of the model used here is:

\[
Y_t = \beta + \Theta(L)C_t + u_t
\]

\[
D(L)u_t = \epsilon_t
\]

\[
\Psi(L)C_t = \delta + \eta_t
\]

where \( Y_t \) is a vector of the logarithm of observed coincident economic variables, \( \beta \) is the mean of \( Y_t \), \( C_t \) represents the logarithm of the state of the economy at time \( t \), \( L \) denotes the lag operator and \( \Theta(L), D(L), \Psi(L) \) are respectively vector, matrix and scalar lag polynomials. The error term \( u_t \) is serially correlated and its dynamics are specified by an autoregressive process \( D(L)u_t = \epsilon_t \) where \( D(L) = 1 - d_1 L - \ldots - d_p L^p \), while the error terms \( \{ \epsilon_t, \eta_t \} \) are assumed to be serially uncorrelated with a zero mean and a diagonal covariance matrix \( \Sigma \).

The stochastic dynamic of \( C_t \) is described by \( \Psi(L)C_t = \delta + \eta_t \) where \( \Psi(L) \) is an autoregressive stationary operator of order \( p \) and \( \delta \) is the mean of \( C_t \). \( Y_t \) is a non-stationary series and it’s possible that \( Y_t \) and \( C_t \) have common stochastic trends. Hence, consider the model in first difference form:

\[
\Delta Y_t = \beta + \Theta(L)\Delta C_t + u_t
\]

\[
D(L)u_t = \epsilon_t
\]

\[
\Psi(L)\Delta C_t = \delta + \eta_t
\]

The coincident index is the estimated value of \( \Delta C_t \) conditional on the information available at time \( t \) and notice \( \Delta C_t \). Then, the indicator is a linear combination of past and present values of \( \Delta Y_t \) variables, that is, \( \Delta C_t = W(L)\Delta Y_t \), where \( W(L) \) is a weighting vector.

Two further steps are necessary for the estimation of the coincident indicator: (i) rewrite Expression (4) and (6) in a state-space form and estimate the parameters of the model and the unobserved state of the economy using a Kalman filter (see Y. Liu (2001) and Appendix 1) and; (ii) in the procedure, each coincident economic variable in the vector \( Y \) is first difference and normalised by subtracting its mean difference and then dividing by the standard deviation of its
difference. Hence, $\Delta C_t$ must be de-normalised and de-logged in order to find the final coincident index.

Finally, to estimate the leading indicator Stock and Watson (1989) used the Vector Autoregressive (VAR) methodology. Formally,

$$
\Delta C_t = \mu_t + \lambda_1(L)\Delta C_{t-1} + \lambda_2(L)X_t + V_t
$$

(7)

$$
X_t = \mu_t + \lambda_1(L)\Delta C_{t-1} + \lambda_2(L)X_{t-1} + V_t
$$

(8)

where $X_t$ is a vector with stationary leading indicators and $V_t$ and $V_{t-1}$ are serially uncorrelated error terms. $\Delta C_t$ is the coincident index.

6. Construction of a Coincident and Leading Indicator with the Stock and Watson methodology

The Coincident Indicator

The first step in estimating a composite index of coincident economic indicators (CEI) is to determine a reference series for the state of the economy. Real GDP was chosen and its chronology developed in the fourth section of this paper. Next, chose indicators in order to determine the state of the economy: this paper uses the industrial production index (IPI), the retail value added (RV) and manufacture value added (manu) as indicators. Why these series? Not only are readily available and account for significant activities in the Barbadian economy, but these series are highly correlated and closely mimics the reference series (see Graphs 3 and 4 and Table 2).
The next step is to estimate Equations (4) to (6). To start, the series are tested for the presence of unit roots and cointegration. The results of the Augmented Dickey and Fuller’s unit root test (see appendix 2) indicate that the log series need to be difference once to be stationary. Moreover, because of the seasonality in the series, the standard X12 procedure developed by the U.S. Census Bureau is used to seasonally adjust the series (see Graph 5). Furthermore, the Johansen’s cointegration test indicates that the series are not co-integrated (rank equals 1) at the 5% level of significance (see Appendix 2).

Now, the Kalman filter, which consists of a sequence of prediction and update steps, can be applied to Equations 4 to 6. Assuming that $C_t$ follows an $AR(1)$ and $u_t$ an $AR(2)$, we obtain the measurement and transition equation (see appendix 3).

The results indicate that a few of the coefficients are statistically significantly different from zero suggesting that the model is reasonably well specified.

The coincident indicator can be written as a function of its components in the following way:

$$\Delta C_{t+1} = W(L)\Delta Y_t$$

where $W(L)$ is a weighting vector that gives the contribution of each variable to the composition of the index. Doing this gives:

$$\Delta C_t = 2.50 \Delta \text{MANU}_t + 3.25 \Delta \text{DIPI}_t - 0.67 \Delta \text{DRV}_t$$

From Graph 7, the coincident indicator index displays similar business cycle characteristics of the Barbadian economy as the reference series, real GDP.
The components of the VAR are the leading indicators discussed above plus the coincident indicator, CEI1. Based on various model selection criteria (see Appendix 4), the optimal model of the VAR is with 4 lags. Also, the Johansen cointegration tests indicate no cointegration at conventional significance levels, justifying that a VAR in first differences is appropriate. The results of the VAR are given below.

After estimating the VAR, the equation \( \Delta C_t \) is considered and the leading indicator is obtained as follow:

\[
LEI_{t+2} = -0.38*DCEI1(-1) - 0.43*DCEI1(-2) - 0.232*DCEI1(-3) + 0.22*DCEI1(-4) - 0.003*DFA(-1) + 0.03*DFA(-2) + 0.03*DFA(-3) + 0.01*DFA(-4) + 0.10*DLSV(-1) + 0.09*DLSV(-2) + 0.09*DLSV(-3) + 0.10*DLSV(-4) - 0.03*DM2(-1) - 0.01*DM2(-2) - 0.01*DM2(-3) - 0.03*DM2(-4) - 0.25*DRPI(-1) - 0.07*DRPI(-2) - 0.02*DRPI(-3) + 0.51*DRPI(-4) - 0.0004
\]

LEI1 is an estimation of the growth on two quarters of the coincident index. In addition, it’s clear that the leading indicator is not the growth rate of GDP but only a way to know if the economy will be in recession or contraction. The graph 9 shows the lag between the growth of LEI1 and the growth of GDP.

Graph 9: Growth of the leading indicator and growth GDP

In order to construct the leading indicator index, a VAR is undertaken using Equations 7 and 8. As is customary, the variables are first checked for stationarity. The results in the Appendix 2 indicate that the variables are all stationary in first difference form. Also, the X12 procedure is utilised to seasonally adjust the data.

The Leading Indicator

The approach for estimating the leading indicator is similar to that for the coincident indicator. All possible available series from different sectors of the economy are considered but only four series are selected: the retail price index (RPI); the net foreign assets of commercial banks (FA); long stay visitors (LSV) and money supply (M2). Graph 8 and Table 3 show a relatively close association of these series with GDP. In order to obtain the evolution of LEI1, the same operation that was done for the CEI1 is repeated. Graph 10 shows the evolution.
In conclusion, the leading indicator is shown as two quarters ahead forecast. It’s not possible to see clearly if the leading indicator predicts the GDP well. An example using a small sample is depicted in Graph 11 and makes the picture clearer.

It can be seen that some peaks and troughs are predicted by the leading indicator. An evaluation for post sample period of 2004 is given in Graph 12. The results indicate that the LEI1 forecasts a peak in 2003:3 for GDP which actually peaks in 2004:1 we have a peak. Then LEI1 is realized two quarters before the change in GDP.

7. A Comparison with Mongardini and Saadi-Sedek Method

The coincident index

Mongardini and Saadi-Sedik (2003), hereafter MS, provides a simplification of the SW method, and they argued that it could be y when there is a limited sample size. It assumes that the reference series are highly correlated with GDP and have a similar evolution. The same indicators in the SW method are utilised here but only industrial production has these features. Hence, a reduced form equation is estimated as follows:

\[
\Delta IP_t = \alpha + \beta \Delta CLI_t + u_t \\
\]

\[
u_t = \epsilon_t + \theta \epsilon_{t-1}
\]

(9)

It’s a simple regression of the reference series on other indicators that represent the state of the economy. \(\Delta IP_t\) is the growth rate of the seasonally adjusted industrial production index, \(\Delta CLI_t\) is a vector of the seasonally adjusted coincident indicators expressed in growth rates, \(u_t\) is an error term with a moving average component of order 1. As the error term is not normally distributed in the regression, the standard errors and covariance matrix are estimated using the Newey-West heteroskedastic-consistent procedure. The results of this estimation are given in Table 4 below.
The final step is to derive the CEI2 from the regression results. As in the SW approach, a simple procedure is used to derive the index: the initial value of the index is set equal to the equivalent observation for industrial production; subsequent observations are then derived by multiplying the previous observation by the fitted quarterly growth rate. The Coincident Indicator Index so derived is shown in Graph 14:

The index seems to represent the state of the economy relatively well. All the turning points in the cyclical GDP are predicted by the CEI2.

The Leading index

As in the estimation of CEI2, the economic activity variable is proxied by the IPI. The statistical relationship is then formulated in the form of the following reduced form equation:

$$\Delta PI_{t+2} = \alpha + \beta \Delta LLI_t + u_t$$

(10)

where $\Delta PI_{t+2}$ is the growth rate of the seasonally adjusted IPI two quarters ahead, $\beta LLI_t$ is a vector of seasonally adjusted leading indicators and $u_t$ is an error term with a moving average
component of order 1. The procedure to estimate equation (10) is the same as that used to determine CEI2 and LEI1 above.

Table 5: Estimation of Leading Economic Indicators

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>DLSV</td>
<td>-0.058389</td>
<td>0.022191</td>
<td>-2.631271</td>
<td>0.0097</td>
</tr>
<tr>
<td>DM2</td>
<td>0.082828</td>
<td>0.055830</td>
<td>1.483563</td>
<td>0.1407</td>
</tr>
<tr>
<td>DRI</td>
<td>0.083570</td>
<td>0.145390</td>
<td>0.582462</td>
<td>0.5623</td>
</tr>
<tr>
<td>DFA</td>
<td>0.066224</td>
<td>0.017762</td>
<td>3.726712</td>
<td>0.0000</td>
</tr>
<tr>
<td>MA(1)</td>
<td>-0.442295</td>
<td>0.061616</td>
<td>-7.216916</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Unlike for CEI2, most of the variables in the estimation (see Table 5) are not statistically significant. However, the fitted value and the growth of IPI appear to be highly correlated (see Graph 15). Hence, it seems possible to construct the index with this simplified method as done above for the more sophisticated SW. The results are given in Graph 16.

Graph 15: Estimation of LEI2: Actual, fitted and residual values

Graph 16: Industrial Production and Leading Indicator
(LEI2 lagged two quarters forward)

Again the two graphs don’t have the same evolution since the leading index just shows the direction of possible changes in the economy. Graph 17 gives a better view. For the period 2000 to 2003, one can see that when the LEI2 predicts the increase in 2000:3, the same increase is true in 2000:2 for the leading index.

Graph 17: Prediction of the LEI2 from 2000 to 2003

In conclusion, this method can provide a good estimation of the future evolution of the economy. It’s simple and clear.

We can see that some peaks and troughs are predicted by the leading indicator. An evaluation for 2004 is given in graph 18:
The results of forecasting: the LEI2 forecasts a peak in 2003:3 for GDP and we see that in 2004:1 we have a peak. Then the prediction two quarters before of the LEI1 is realized two quarters after.

8. Conclusion
This paper has attempted to provide a structured approach to analyzing business cycles in Barbados. The models developed are based on the single-index methodology of Stock and Watson and they gave coincident indices that dated and followed the Barbados business cycles closely. The model also established leading indicators which could be used to predict future movements in the coincident index. However, it’s possible that there indices could be untamed with the availability of more highly correlated data and on a higher frequencies for example monthly.

References

Bergeron Luc, Yvon Fauvel and Alain Paquet; “Application de la méthode de STOCK et WATSON pour la construction d’un indicateur avancé synthétique de l’économie canadienne”.


**Appendix 1: State space model and Kalman filter**

Space State model

Many time-series models used in econometrics are special cases of the class of linear state space models developed by engineers to describe physical systems. The Kalman filter, an efficient recursive method for computing optimal linear forecasts for such models, can be exploited to compute the exact Gaussian likelihood function.

The linear state-space model postulates that an observed time series is a linear function of a (generally unobserved) state vector and the law of motion for the state vector is first-order vector autoregression. More precisely, let \( y_t \) be the observed variable at time \( t \) and \( \mathbf{x}_t \) denote the values taken at time \( t \) of a vector of state variables. Let \( \mathbf{A} \) and \( \mathbf{B} \) be \( p \times p \) and \( p 	imes s \) matrices of elements. We assume that \( \{ \mathbf{y}_t \} \) is generated by:

\[
\begin{align*}
\mathbf{y}_t &= \mathbf{F}\mathbf{y}_{t-1} + \mathbf{B}\mathbf{x}_t \\
\mathbf{x}_t &= \mathbf{A}\mathbf{x}_{t-1} + \mathbf{w}_t
\end{align*}
\]

where the vector \( \mathbf{y}_t \) and the vector \( \mathbf{x}_t \) are mean-zero, white-noise processes, independent of each other and of the initial value \( \mathbf{x}_0 \). We denote \( \mathbf{F} = \mathbb{E}(\mathbf{y}_t | \mathbf{y}_{t-1}) \) and \( \mathbf{E} = \mathbb{E}(\mathbf{w}_t | \mathbf{y}_{t-1}) \). Equation (1) is sometimes called the “measurement” equation while (2) is called the “transition” equation. The assumption that the autoregression is first-order is not restrictive, since higher-order systems can be handled by adding additional state variables.

The Kalman Filter

Denote the vector \( \{y_0, \ldots, y_t\} \) by \( \mathbf{y}_t \). The Kalman filter is a recursive algorithm for producing optimal linear forecasts of \( y_{t+1} \) from the past history \( \mathbf{y}_t \), assuming that \( \mathbf{A} \), \( \mathbf{B} \), \( \mathbf{C} \), and \( \mathbf{D} \) are known. Define

\[
\begin{align*}
\delta_t &= \mathbb{E}(\mathbf{y}_t | \mathbf{y}_{t-1}) \\
\Sigma_t &= \mathbb{V}(\mathbf{y}_t | \mathbf{y}_{t-1})
\end{align*}
\]

If the \( \mathbf{w}_t \) and \( \mathbf{v}_t \) are normally distributed, the minimum MSE forecast of \( y_{t+1} \) at time \( t \) is \( \delta_{t+1} \). The key fact (which we shall derive below) is that, under normality, \( \delta_{t+1} \) can be calculated recursively by:

\[
\begin{align*}
\delta_{t+1} &= \mathbf{A}\delta_t + \mathbf{B}\mathbf{x}_t \\
\Sigma_{t+1} &= \mathbf{A}\Sigma_t \mathbf{A}' + \mathbf{B}\mathbf{B}'
\end{align*}
\]

starting with the appropriate initial values \( \{\delta_0, \Sigma_0\} \). To forecast \( y_{t+1} \) at time \( t \), one needs only the current \( \mathbf{y}_t \) and the previous forecast of \( \delta_t \) and its variance. Previous values \( \mathbf{y}_0, \ldots, \mathbf{y}_{t-1} \) enter only through \( \delta_t \). Note that \( y_t \) enters linearly into the calculation of \( \delta_t \) and does not enter at all into the calculation of \( \Sigma_t \). The forecast of \( y_{t+1} \) is a linear filter of previous \( y_t \). If the errors are not normal, the forecasts produced from iterating (4) are still of interest; they are best linear predictions.

---

More information is available on [http://emlab.berkeley.edu/~rothenbe/Fall2004/kalman.pdf](http://emlab.berkeley.edu/~rothenbe/Fall2004/kalman.pdf)
Appendix 2: Unit Root and Cointegration test results for the coincident and leading indicator variables

Table 6: Augmented Dickey-Fuller test statistic

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<th>Level</th>
<th>First difference</th>
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<td>(0.000)</td>
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<td>(0.000)</td>
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<td>(0.458)</td>
<td>(0.000)</td>
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<td>(1.000)</td>
<td>(0.000)</td>
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</table>

Numbers in brackets are the probability of the p-value
The critical test at 5% is -2.887
Appendix 3: Kalman Filter Results

Measurement equation

\[ \Delta \text{MANU} = -3.52 - 1.15\Delta \text{C}_t + U_t \]
\[ (1.00) \quad (0.95) \]
\[ \Delta \text{DIPI} = -0.13 - 3.43\Delta \text{C}_t + U_t \]
\[ (1.00) \quad (0.00) \]
\[ \Delta \text{DRV} = -12.50 - 0.61\Delta \text{C}_t + U_t \]
\[ (1.00) \quad (0.50) \]

State equations

\[ U_{\text{MANU}}^t = -0.57U_{\text{MANU}}^{t-1} - 0.79U_{\text{MANU}}^{t-2} + \epsilon_{U_{\text{MANU}}}^t + \epsilon_{\text{MANU}}^t \quad \sigma_{\text{MANU}} = -3.08 \]
\[ (0.00) \quad (0.00) \quad (0.00) \]
\[ U_{\text{DIPI}}^t = -0.08U_{\text{DIPI}}^{t-1} - 15.04U_{\text{DIPI}}^{t-2} + \epsilon_{U_{\text{DIPI}}}^t \quad \sigma_{\text{DIPI}} = -5.23 \]
\[ (0.00) \quad (0.13) \quad (0.00) \]
\[ U_{\text{DRV}}^t = 0.52U_{\text{DRV}}^{t-1} - 1.34U_{\text{DRV}}^{t-2} + \epsilon_{U_{\text{DRV}}}^t \quad \sigma_{\text{DRV}} = -2.45 \]
\[ (0.00) \quad (0.01) \quad (0.00) \]
\[ \Delta \text{C}_{\text{MANU}}^t = 0.76\Delta \text{C}_{\text{MANU}}^{t-1} + \eta_\text{c} \quad \sigma_\text{c} = 1 \]
\[ (0.09) \]
\[ \Delta \text{C}_{\text{DIPI}}^t = 0.95\Delta \text{C}_{\text{DIPI}}^{t-1} + \eta_\text{c} \quad \sigma_\text{c} = 1 \]
\[ (0.001) \]
\[ \Delta \text{C}_{\text{DRV}}^t = 0.36\Delta \text{C}_{\text{DRV}}^{t-1} + \eta_\text{c} \quad \sigma_\text{c} = 1 \]
\[ (0.00) \]

*Numbers in brackets are standard errors

In order to find the expression of \( C_t \), we de-normalize \( Y_t \)

For the first equation we have:

\[
\left( \frac{\Delta \text{MANU} + 3.52}{-3.08} \right) \times \frac{1}{1.15} = \Delta \text{C}_t = 0.05\Delta \text{MANU}
\]

and

\[ \Delta \text{C}_t = 0.025\Delta \text{DIPI} \]
\[ \Delta \text{C}_t = 0.67\Delta \text{DRV} \]

Then:

\[ \Delta \text{C}_t = 0.050\Delta \text{MANU} + 0.025\Delta \text{DIPI} + 0.67\Delta \text{DRV} \]
Appendix 4: Optimal VAR lag and VAR output

Sample(adj usted): 1975-2 2003:4
Included observations: 115 after adjusting endpoints
Trend assumption: Linear deterministic trend
Series: RPI M2 FA LSV
Lags interval (in first differences): 1 to 4

Hypothesized No. of CEC(s) Max-Eigen Value 5 Percent Critical Value 1 Percent Critical Value
None 0.195451 25.09941 27.07 32.24
At most 1 0.146509 18.21834 20.97 25.52
At most 2* 0.138499 17.14413 14.07 18.63
At most 3 0.008235 0.950958 3.76 6.65

*(**) denotes rejection of the hypothesis at the 5%(1%) level
Max-eigenvalue test indicates no cointegration at both 5% and 1% levels

VAR Lag Order Selection Criteria
Endogeneous variables: DCEI DFA DLSV DRPI
Exogeneous variables: C
Date: 07/21/05 Time: 07:14
Sample: 1974:1 2003:4
Included observations: 111

Vector Autoregression Estimates
Sample(adj usted): 1975-2 2003:4
Included observations: 115 after adjusting endpoints
Standard errors in ( ) & t-statistics in [ ]

DCEI DFA DLSV DM2 DRPI
DCEI(-1) -0.189674 0.913121 0.479308 0.196424 0.048424
(-0.01027) (0.06347) (0.20697) (0.21011) (0.00179)
[-3.86069] [1.36263] [2.20252] [1.38291] [0.48460]
DCEI(-2) -0.428534 -0.973600 0.364238 0.108291 -0.566415
(-0.16793) (0.72019) (0.22808) (0.21317) (0.00449)
[-0.60683] [-1.05625] [1.45125] [0.48409] [-2.17052]
DCEI(-3) -0.232496 0.555301 0.221036 0.109952 -0.005798
(-0.10786) (0.73597) (0.22513) (0.22024) (0.00450)
[-2.13520] [0.77452] [0.89189] [0.83997] [-0.03785]
DCEI(-4) 0.292214 -0.663617 0.270667 0.018892 0.017042
(-0.10559) (0.71690) (0.22015) (0.21144) (0.00440)
[-2.17868] [0.91915] [1.22910] [0.66541] [0.38728]
DFA(-1) 0.003970 -0.616177 0.029285 -0.012124 0.002948
(-0.01470) (0.10017) (0.03066) (0.03078) (0.00612)
[-0.23607] [-0.15120] [0.92569] [0.68966] [0.48136]
DFA(-2) 0.038163 -0.204704 0.017092 -0.004832 -0.006148
(-0.01649) (0.11240) (0.05435) (0.04345) (0.00967)
[-2.31412] [-1.82194] [1.07875] [-1.39928] [-0.49450]
DFA(-3) 0.002531 -0.263640 -0.004685 -0.001419 -0.009863
(-0.01649) (0.11240) (0.05435) (0.04345) (0.00967)
[-2.31412] [-1.82194] [1.07875] [-1.39928] [-0.49450]
DFA(-4) 0.016184 -0.121233 -0.00407 -0.017824 -0.003588
(-0.01618) (0.09753) (0.02983) (0.02997) (0.00956)
[1.63866] [2.20221] [1.63633] [-0.20402] [-1.43446]
DLSV(-1) 0.105842 -0.254959 -0.397391 0.21067 0.00790
(-0.04147) (0.28263) (0.08846) (0.08804) (0.00728)
[-1.25526] [-2.09211] [-1.59560] [-0.91938] [-0.15197]
DLSV(-2) -0.087884 -0.507086 -0.351680 0.06842 0.025219
(-0.04110) (0.28083) (0.08870) (0.08868) (0.00713)
[-1.21662] [-1.85996] [-1.40732] [0.99254] [1.47228]
DLSV(-3) 0.087698 -0.293103 -0.321170 0.107069 0.011269
(-0.03956) (0.26907) (0.08286) (0.08260) (0.00814)
[-2.23248] [-1.83124] [-1.39383] [1.30326] [1.13177]
DLSV(-4) 0.097964 -0.205699 0.156743 0.064403 0.017170
(-0.03839) (0.26165) (0.08000) (0.08040) (0.00810)
[-2.54690] [-1.87615] [-1.46058] [-0.80105] [-0.16698]
AIC Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
DCEI DFA DLSV DM2 DRPI
DCEI(-1) 0.027237 0.641645 0.012714 -0.064981 -0.002043
(-0.03010) (0.34814) (0.01649) (0.01697) (0.00129)
DCEI(-2) -0.010296 0.617773 -0.139864 -0.416300 0.050927
(-0.06186) (0.41617) (0.12731) (0.12788) (0.02654)
[-1.44841] [-0.99662] [-0.70507] [-0.60134]
DCEI(-3) -0.014811 -0.229125 -0.168620 -0.242182 0.018555
(-0.06100) (0.41575) (0.12718) (0.12772) (0.02542)
[-0.10623] [-0.25656] [-0.16897] [-0.16870] [0.14658]
DCEI(-4) -0.029399 0.104790 -0.211440 -0.099420 0.029541
(-0.03137) (0.35013) (0.10711) (0.10759) (0.02241)
[-0.57055] [-0.29992] [-2.25140] [-0.24210] [0.13796]
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| Determinant Residual Covariance | 9.98E-13 |
| Log Likelihood (d.f. adjusted) | 802.4737 |
| Akaike Information Criteria | -12.1293 |
| Schwarz Criteria | -9.623736 |