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“Throwing Darts and Missing The Bullseye: Fiscal and Debt Sustainability In Open Economies”

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Throwing Darts and Missing the Bullseye: Fiscal and Debt Sustainability in Open Economies*

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Abstract

The standard debt sustainability condition emerges from the government's intertemporal budget constraint and omits goods market (flow) equilibrium. Consequently, the debt-targeting fiscal rule of a long-run primary surplus overshoots/undershoots the steady-state debt ratio. In other words, the standard debt sustainability analysis is not stock-flow consistent, which is necessary for appropriate analysis as debt ratios combine stock (debt) and flow (GDP or exports) variables. This research shows that the price of stock-flow inconsistency is significant volatility: debt or foreign exchange crises. This article formalises a stock-flow consistent model of fiscal and debt sustainability in an open economy. It demonstrates that a primary deficit as a share of GDP obtains goods market equilibrium at potential output and a steady-state debt ratio, irrespective of the exchange rate regime and even when the economy is dynamically efficient. The model derives a simple rule that specifies the precise primary deficit required for stock-flow equilibria.

JEL Classification: E32, E62, F31, F41, H62, H63

Keywords: Debt Sustainability, Primary Deficit, Stock-Flow Equilibria, Open Economy

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1 Introduction

The fiscal response to COVID-19 in both developed and developing economies has raised the old question on public finance: What determines fiscal and public debt sustainability? The Domar debt sustainability condition presents the standard answer: The real interest rate-growth rate inequality $\left(r-g \lesseqgtr 0\right)$. Based on this condition, when the long-term real interest rate r is less (greater) than the long-run growth rate g, only a primary fiscal deficit (surplus) can stabilise the public debt ratio (Blanchard 2019; Domar 1944). The long-run expectation is that dynamic efficiency holds ($r > g$), where a long-run primary surplus as a share of GDP stabilises the debt-GDP ratio if it accounts for predictable increases in entitlement spending (Cerniglia et al. 2021)—this is the standard approach to fiscal and public debt sustainability.

This article demonstrates that the Domar debt sustainability condition is stock-flow inconsistent and induces significant volatility: foreign exchange and debt crises. The stock-flow inconsistency emerges because the basis of the Domar condition is the government's budget constraint, which only identifies the *sources* of deficit spending: bonds, foreign assets, and money creation. This approach omits two key factors: 1. The *use* of fiscal policy or the requirements of internal and external balance to stabilise the goods market at full employment, and 2. The *endogenous* source of high-powered money (HPM).

This paper develops an open economy and stock-flow consistent model in continuous time to account for these omissions and derive a primary balance that stabilises the debt-GDP ratio and goods market equilibrium at potential output (the bullseye). Public debt refers to the sum of domestic and foreign-currency-denominated bonds issued by the central government. Unlike the standard approach, the model starts from the goods market-clearing condition and introduces financial stocks that account for income flows related to internal and external sectoral balances. Given this starting point, bank credit is a source of domestic demand and (endogenous) high-powered money, so the latter remains crucial to debt analysis. However, similar to the standard approach, the model does not permit monetisation of the fiscal deficit (the *exogenous* source of high-powered money) on the grounds of exchange rate stability and maintaining a credible peg. The model also accounts for several stylised facts: rich households and institutional investors hold public debt, banks operate in the primary market in low-income countries, and the Original Sin holds.[1]

[1] Original Sin is a term coined by Barry Eichengreen, Ricardo Hausmann, and Ugo Panizza to refer to a situation where developing economies are not able to borrow on the world market in their domestic currency.

The stock-flow model turns the Domar condition on its head. In both fixed and flexible exchange rate systems, a primary deficit as a share of GDP stabilises the public debt-GDP ratio (b) unless the *weighted* real interest rate-growth rate inequality is implausibly large.

$$\dot{b} = \overbrace{\Omega}^{\text{Primary Deficit}} - \underbrace{\overbrace{\delta\omega_1 b_p}^{\text{Private Debt}} + \overbrace{\delta h}^{\text{HPM}}}_{\text{Negative Net Effect}} - \overbrace{\gamma f_{cb}}^{\text{CB's Foreign Assets}} + \overbrace{\Big[(r + \gamma\rho_1\alpha) - g\Big]b}^{\text{Weighted Interest Rate-Growth Rate Inequality}}$$

In the case of a fixed peg, the model above produces an augmented condition for dynamic efficiency ($r + \gamma\rho_1\alpha > g$) as it accounts for the share of external debt in total government debt, α. When the latter rises, more foreign assets service external debt obligations, which reduces the stock of foreign assets used for private consumption and investment. Thus, the private sector accumulates public sector domestic debt as a form of *forced savings*.[2]

An increase in the share of foreign assets held by the central bank decelerates the public debt ratio as it activates the *demand constraint* or reduces the long-term demand for government bonds *à la* Sargent and Wallace (1981). Two key mechanisms are at play. First, the government becomes a *net lender* in world markets or repays its debt.[3] Second, a higher share of foreign assets means that the private sector can import consumer and capital goods without compromising the peg. So, firms *retire* their stocks of public debt to accumulate foreign assets and their demand constraint for government bonds binds. Public debt is retired because the Original Sin holds, poor households are too poor to hold securities, and firms cannot sell bonds to rich households who own the firms. Also, households or firms reduce their bank deposits to purchase foreign currencies from commercial banks, and fewer bank deposits activate the demand constraint in emerging and advanced economies. In low-income countries, fewer bank deposits mean that commercial banks exchange the corresponding excess bank reserves for foreign assets.[4] Banks purchase foreign assets instead of low-yielding government bonds in the primary market because fewer bank deposits also mean excess bank liquidity, which is inconsistent with profit maximisation and portfolio diversification.[5]

[2] See Khemraj (2009) for evidence that foreign exchange shortages force commercial banks to hold government securities in the case of Guyana.

[3] This channel is consistent with the Guidotti-Greenspan rule, where a country's reserves should cover its short-term external debt, and a rich empirical literature that underscores the precautionary motivates of reserve accumulation: insurance against sudden stops to repay external debt, among others (Durdu et al. 2009; Aizenman and Lee 2007).

[4] It is a stylised fact that excess bank reserves lead to capital flight (Flood and Garber 1984; Krugman 1979). This is widely documented since the start of quantitative easing by the Federal Reserve and the European Central Bank (Dedola et al. 2021), and in earlier studies on developing economies (Khemraj 2014: 47).

[5] This claim holds in the case of effective monetary policy, where there is no monetisation or fiscal dominance.

It is worth noting that the central bank cannot serve as the residual purchaser of government securities and maintain a credible peg.

Private debt as a share of GDP also activates the demand constraint. (i) When corporate debt rises, firms retire public debt to repay their domestic liabilities and their demand constraint binds.[6] (ii) The same holds if households or firms repay their debt with bank deposits as noted previously. Also, when banks extend credit, they demand high-powered money to facilitate inter-bank clearing and satisfy the demand for cash. An increase in the share of high-powered money accelerates the public debt ratio because it raises *voluntary savings* in the form of government bonds. The transfer of assets from the debtor to the creditor increases savings, so high-powered money reflects the rise in wealth that accrues to creditors (rich households) or shareholders of the banks.[7] Note carefully that a credible peg requires that the government satisfies the demand for government bonds as rich households may purchase foreign securities instead.

A recent study documents that the average size of foreign assets held by central banks as a share of GDP is 30% in emerging markets in 2018 (Arslan and Cantu 2019). Moreover, it is a stylised fact that private sector debt as a share of GDP ranges from 50% to more than 100% in advanced and emerging economies. Further, efficient bankers minimise their stock of high-powered money, and foreign currency risk encourages governments and markets to impose a hard limit on the share of external debt in total government debt. Therefore, a primary deficit stabilises the debt ratio under reasonable values for r, α, and g.

The model also demonstrates that the exogenous increase in high-powered money (monetisation) accelerates the debt ratio by reducing the stock of foreign assets held by the central bank and increasing high-powered money.[8] Further, in contrast to the Domar model, the stock-flow model converges faster to its steady-state debt ratio. The risk of currency mismatch caps the share of external debt in total public debt and explains this result.[9]

However, banks are inundated with excess reserves and liquidity under fiscal dominance and maximise profits by increasing the interest rate spread (Constantine 2022).

[6] The literature suggests that public debt crowds out corporate debt but the reverse channel is advanced here (Lugo and Piccillo 2019; Demirci et al. 2019).

[7] See Mian et al. (2021) for evidence that American households in the Top 1% are the principal holders of government securities between 1982 to 2016, and consult Bouis (2019) and Mihaljek et al. (2002) for verification that commercial banks and other financial institutions hold the largest proportion of domestic bonds in developing economies.

[8] See footnote 3 for evidence.

[9] The faster speed of convergence is consistent with the evidence of low debt tolerance in developing countries and emerging markets (Reinhart et al. 2003).

The analogous version of the stock-flow model for the case of a pure float shows that a primary deficit stabilises the debt ratio if the private sector's domestic debt as a share of GDP exceeds the sum of high-powered money as a share of GDP, and the weighted real interest rate-growth rate inequality. In this case, the weighted real interest rate rises with the long-run rate of nominal depreciation and provides for a faster speed of convergence to its long-run debt ratio as compared to the Domar model. The key difference between this result and the case of a fixed peg is that monetisation depreciates the nominal exchange rate and accelerates the debt ratio by raising the local currency burden of external liabilities.[10]

This paper contributes to several related literature. First, the standard approach to debt sustainability builds on the Domar model and indicates that the government's initial domestic (external) debt stock must equal the net present value of its future primary fiscal (trade) surplus (Blanchard and Weil 2001). In plain terms, the government's intertemporal budget constraint (IBC) must be satisfied for debt sustainability. Blanchard and Weil present a stronger version of the Domar model: even if $r < g$, public debt cannot be permanently rolled over when there is uncertainty and public debt fails to provide intergenerational insurance. However, the standard approach violates stock-flow consistency and implies that a fiscal rule derived from the IBC does not guarantee goods market equilibrium at full capacity. In other words, it is not sustainable from the perspective of goods market flow equilibrium. More recently, Blanchard and Das (2017) posit that debt sustainability analyses omit the fact that the present value of net exports is a random variable, and there is always an exchange rate depreciation that makes external debt sustainable (assuming an expansionary currency depreciation). The stock-flow model also derives similar results when a nominal depreciation is expansionary. However, newly emerging evidence suggests that expansionary currency depreciation is the exception rather than the rule (Gopinath et al. 2020; Serana and Sousa 2017). In this case, the model shows that a contractionary depreciation misses the bullseye if the basis of the fiscal rule is the IBC.

Second, only a handful of studies have highlighted and resolved the stock-flow inconsistency of the debt-GDP indicator (Canofari et al. 2020; Godley and Lavoie 2007). Canofari and co-authors scale public debt by the stock of wealth to resolve the inconsistency. However, their model fails to do better than the Domar condition as the government's IBC yields the fiscal rule. In contrast, Godley and Lavoie show by way of simulation that a long-run fiscal deficit is consistent with

[10] See Fisera et al. (2021) for recent evidence of this channel in a panel of 41 emerging economies over the years 1999–2019.

stable debt-GDP and trade deficit ratios. Unlike the latter study, the present work also accounts for internal balance and shows that the same result holds irrespective of the exchange rate regime. This literature is closely related to the scholarship on long-run fiscal deficits and public debt. For example, Krugman (2020) and Aspromourgos et al. (2010) propose the idea of a permanent fiscal deficit based on the prediction that dynamic inefficiency holds in the long-run and that the monetary authority influences the long-term yield on government bonds, respectively. The stock-flow model presents sharper results: long-run fiscal deficits are sustainable even if dynamic efficiency holds and the real interest rate is independent of monetary policy. Mian et al. (2022) and Reis (2021) are two recent studies that qualify the conditions of a permanent fiscal deficit. Mian et al. formally demonstrate that the interest rate rises with the debt stock, so primary deficits are only sustainable up to some threshold. Reis takes a different approach and shows that permanent fiscal deficits pay for themselves when a bubble premium exists, $m > r$, where m is the marginal product of capital. In Reis' model, primary deficits raise the bubble premium to some threshold, after which the bubble pops due to a fall in r and contracts the demand for sovereign debt. The key difference between these studies and the present article is that they do not admit stock-flow consistency.

Third, closed economy models show that monetisation increases the inflation rate and reduces the real debt burden (Buiter et al. 1985). In contrast, the stock-flow model demonstrates that monetisation accelerates the public debt ratio as it reduces the stock of foreign assets and depreciates the nominal exchange rate. Fourth, resource availability determines the extent of fiscal space according to the established view—foreign assets that exceed three months of import cover (Worrell 2015)—or the difference between the country's current debt level and its debt limit (Ghosh et al. 2013). However, the requirements of stock-flow equilibria prove that the degree of resource utilisation in the goods market also determines fiscal space. Fifth, this study shows that only a stock-flow consistent fiscal rule stabilises the debt ratio and goods market at a constant rate of nominal depreciation. Ergo, the necessity of foreign exchange intervention by inflation-targeting central banks may be the price of stock-flow inconsistent fiscal rules (Ghosh et al. 2016). Sixth, the article contributes to the literature on optimal reserve balances (Obstfeld et al. 2010; Jeanne and Ranciere 2009; Worrell 1976). It shows that stock-flow equilibria determine optimality, and the model nests many of the established drivers of reserve accumulation. Finally, the stock-flow model violates the Tinbergen Rule, which recommends that the policymaker utilises n instruments to obtain n targets. The paper demonstrates that the Tinbergen Rule produces economic cycles: exchange rate volatility, foreign exchange, and debt crises in economies with a debt-targeting fiscal rule. Ergo, policymakers must aim for stock-flow consistent fiscal rules for debt, goods market, and exchange rate stability.

The remainder of the paper is organised as follows. Section 2 shows how a debt-targeting fiscal rule misses the bullseye by design, and Section 3 introduces the stock-flow consistent model. Section 4 concludes and omitted proofs are presented in the Appendix.

2 Missing the Bullseye and Debt Crises

This section demonstrates that the Domar debt sustainability condition is not stock-flow consistent, which implies that a debt-targeting fiscal rule induces a debt crisis by *design.*

Domar Debt Sustainability Condition. The government's budget identity is given by:

$$G - T = \dot{B} + \dot{H} - iB, \tag{1}$$

where $G - T$ is the primary balance, $\dot{B}$ and $\dot{H}$ are the time derivative of government debt and high-powered money, respectively, and iB denotes interest payments. Monetisation is ruled-out in the interest of price stability, so $\dot{H} = 0$.

Define the debt-GDP ratio (b) as

$$b = \frac{B}{PY},$$

and the stock of government debt is written as follows.

$$B = (b)PY \tag{2}$$

Substitution of (2) into (1) and rearranging in terms of $\dot{B}$ yields the evolution of the stock of government debt.

$$\dot{B} = G - T + i(b)PY \tag{3}$$

To derive the evolution of the debt-GDP ratio, take the total differential of (2) and divide by nominal GDP:

$$\frac{\dot{B}}{PY} = \frac{\dot{b}PY + \dot{P}bY + \dot{Y}bP}{PY},$$

which simplifies to the following, where $g = \dot{Y}/Y$ is the long-run growth rate and $\pi = \dot{P}/P$ is the rate of inflation.

$$\dot{b} = \frac{\dot{B}}{PY} + b(-\pi - g)b$$

Substitution of Condition (3) into this result yields:

$$\dot{b} = \frac{G-T}{PY} + (i - \pi - g)b.$$

After invoking the Fisher equation ($r = i - \pi$), the dynamic evolution of public debt-GDP ratio is given below:

$$\dot{b} = \frac{G-T}{PY} + (r-g)b, \tag{4}$$

where r is the long-term real interest rate. The following outlines a formal definition of fiscal and public debt sustainability.

Definition 2.1 (Debt and Fiscal Sustainability)**.** *When the government's total public debt as a share of GDP converges to a finite value (b^*) or a steady state, such that, $\dot{b} = 0$, public debt is sustainable. Any fiscal strategy that achieves $\dot{b} = 0$ is sustainable.*

The Domar condition for debt sustainability is summarised in the following Axiom.

Axiom 2.1 (Domar Debt Sustainability Condition)**.** *When $r < g$, the debt ratio converges to a finite value, if and only if, the primary deficit as a share of GDP is equal to $r < g$. Conversely, when $r > g$, the government must incur a long-run primary surplus as a share of GDP equal to $r > g$ to ensure debt sustainability.*

Dynamic Efficiency. The long-run expectation is that $r > g$, which implies that debt sustainability requires a long-run primary surplus as a share of GDP. The Domar debt-targeting fiscal rule follows when $\dot{b} = 0$, and where $\Omega = \frac{G-T}{PY}$ and the subscript DC indicates that the basis of the rule is the Domar condition.

$$\Omega_{DC} = b(g-r) \tag{5}$$

This sustainability condition assumes that the debt-targeting fiscal rule is consistent with goods market equilibrium at full employment. However, goods market equilibrium at full capacity and a long-run primary surplus are only jointly possible with a private sector deficit or external surplus (through sectoral balance accounting). If neither of these requirements is realised, the debt-GDP ratio rises or falls relative to the policy target. The following Proposition summarises this point.

Proposition 2.1 (Domar Condition and Stock-Flow Inconsistency)**.** *The Domar debt sustainability condition or its implied fiscal rule* Ω_{DC} *is not consistent with stock-flow equilibria.*

The basic intuition is that stock-flow equilibria require a fiscal rule consistent with a stable debt-GDP ratio and full employment (flow) equilibrium. Since the Domar fiscal rule omits the external and private sector balances, it may generate excess or deficient aggregate demand. Therefore, there are necessarily upswings and downswings in the trajectory of the debt-GDP ratio if the fiscal authority adheres to the Domar fiscal rule. The latter is akin to *throwing darts and missing the bullseye*, where the bullseye is the government's primary balance that obtains stock-flow equilibria.

The Case of a Fixed Peg. This insight is summarised in the following Proposition for the case of a fixed peg.

Proposition 2.2 (Missing the Bullseye: Fixed Peg)**.** *In a fixed exchange rate regime, the Domar fiscal rule* Ω_{DC} *necessarily overshoots and undershoots the primary fiscal balance and central bank's stock of foreign assets consistent with stock-flow equilibria.*

Figure 1: The Case of a Fixed Peg: Domar's Condition and Missing the Bullseye

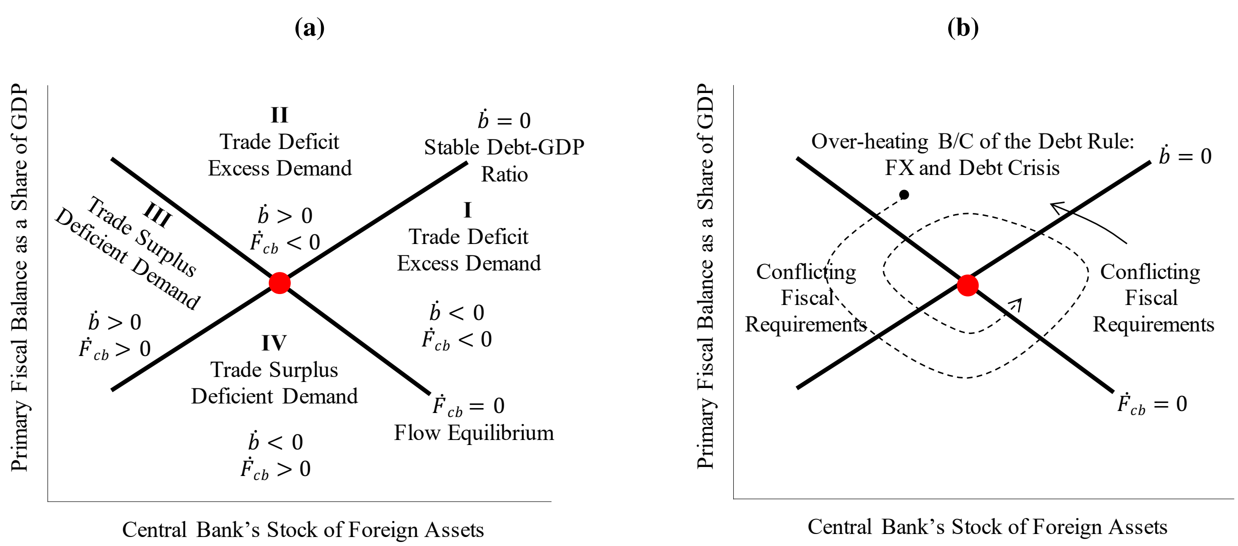


Figure 1 illustrates the basic idea. The upward-sloping locus $\dot{b} = 0$ shows the combinations of the primary fiscal balance and the central bank's stock of foreign assets that are consistent with a stable or steady-state debt ratio. It is upward sloping because as the central bank's stock of foreign assets increases, so does the long-run growth rate, which requires a fiscal deficit to stabilise the public debt ratio. The basic intuition is as follows. As the central bank's stock of foreign assets rises, firms can import intermediate capital goods to accelerate economic growth without compromising the exchange rate peg. In turn, the downward sloping locus $\dot{F}_{cb} = 0$ illustrates flow equilibrium, which depicts the various combinations of the primary fiscal balance and the central bank's stock of foreign assets that are consistent with full employment. It is negatively sloped because a higher stock of foreign assets held by the central bank increases private consumption and investment without compromising the exchange rate peg, which engenders excess demand. Consequently, fiscal austerity is necessary to restore goods market equilibrium. Stock-flow equilibria are realised when $\dot{b} = \dot{F}_{cb} = 0$; the bullseye (red dot).

Figure 1(a) is divided into four quadrants and illustrates the various stock-flow disequilibria. For example, quadrant I illustrates that excess demand engenders a trade deficit and a falling public debt ratio. Note carefully that these disequilibria require a *different fiscal response*—a fiscal deficit is necessary to stabilise the public debt ratio—but fiscal austerity is required to stabilise the goods market. Since only one fiscal strategy can rule the day, cyclical fluctuations are inevitable. It follows that any policy that adheres to the Domar fiscal rule Ω_{DC}, that is, a debt-targeting fiscal rule, produces counter-clockwise adjustments that overshoot and undershoot stock-flow equilibria as is shown in Figure 1(b). This implies that the Domar fiscal rule Ω_{DC} is *pro-cyclical*, which Figure 1(a) verifies after close inspection. For example, a stable debt ratio requires a fiscal deficit in quadrant I when the economy overheats, but fiscal austerity in quadrant III when there is excess capacity. Also, a debt-targeting rule induces a debt crisis as a matter of design. In quadrant I, debt-targeting increases the fiscal deficit to stabilise the debt ratio, which overheats the economy and reduces the stock of foreign assets held by the central bank. In turn, Figure 1(b) shows that this engenders a foreign-currency crisis and undermines the sovereign's ability to service its external debt.

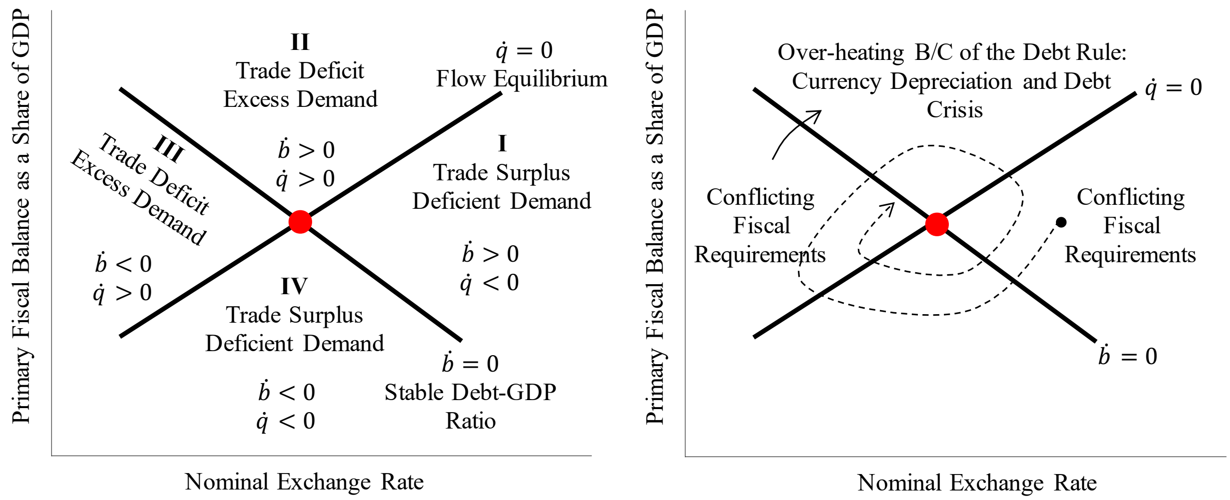
The Case of a Pure Float: Contractionary Depreciation. In this case, a debt-targeting rule provides for an overshooting and undershooting of the stock-flow equilibria consistent with debt and exchange rate stability. The following Proposition summarises this point.

Proposition 2.3 (Missing the Bullseye: Pure Float)**.** *When a nominal currency depreciation is contractionary, the Domar fiscal rule Ω_{DC} necessarily overshoots and undershoots the primary fiscal balance and nominal exchange rate consistent with stock-flow equilibria.*

Figure 2 illustrates the basic idea. The upward-sloping locus $\dot{q} = 0$ shows the combinations of the government's primary fiscal balance and the nominal exchange rate that are consistent with

Figure 2: The Case of a Pure Float: Domar's Condition and Missing the Bullseye

(a) The Case of Contractionary Depreciation **(b)** Clockwise Adjustments



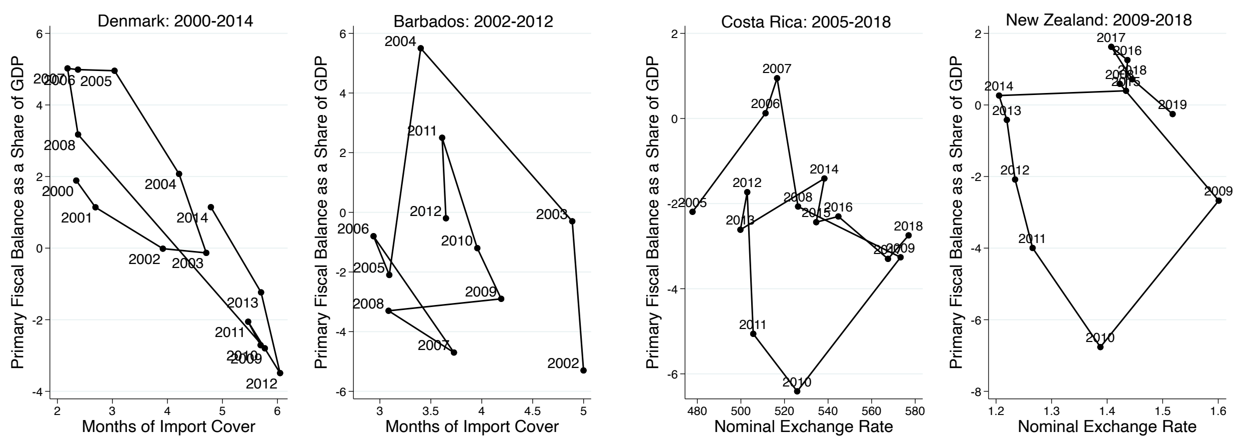
goods market equilibrium at potential output. It is upward sloping because a nominal depreciation reduces aggregate demand (the case of contractionary depreciation) and requires a primary deficit to stabilise the goods market. In the contractionary case, a nominal depreciation reduces the wage share and depresses private consumption or deteriorates the trade balance when domestic substitutes are unavailable or inadequate. In turn, the downward-sloping locus $\dot{b} = 0$ shows the combination of the primary balance and the nominal exchange rate that stabilises the debt-GDP ratio. It is negatively sloped because a nominal depreciation reduces long-run growth and accelerates the public debt ratio, which requires fiscal austerity for debt sustainability. Stock-flow equilibria are realised when $\dot{b} = \dot{q} = 0$.

Figure 2(a) is divided into four quadrants and illustrates the various stock-flow disequilibria. For example, the debt ratio explodes and the nominal exchange rate rapidly appreciates in quadrant I—consistent with deficient aggregate demand and a trade surplus (flow disequilibrium). These disequilibria require a *different fiscal response*—fiscal austerity is necessary to stabilise the public debt ratio—but a fiscal deficit is required to stabilise the goods market and the nominal exchange rate. Consequently, a debt-targeting fiscal rule produces clockwise fluctuations around the stock-flow equilibria as Figure 2(b) demonstrates. In particular, it shows that a debt-targeting rule leads to overheating and rapid depreciation of the nominal exchange rate, which raises the local currency burden of external debt.

Empirical Phase Diagrams. Figure 3(a) uses Denmark and Barbados as examples to present empirical support for the dynamic predictions in the case of a fixed peg. These countries maintain credible pegs and verify the *predicted* counter-clockwise adjustments that overshoot and undershoot stock-flow equilibria. Denmark appears to have completed a cycle with primary surpluses in the early 2000s, but a second cycle of primary deficits is evident in the later years. The data shows that primary deficits dominate the cyclical oscillations in Barbados. Figure 3(b) shows the empirical phase diagrams for the case of a pure float, where Costa Rica and New Zealand serve as examples. These countries maintain flexible exchange rates and illustrate the *expected* clockwise adjustments that overshoot and undershoot stock-flow equilibria. Also, primary fiscal deficits dominate the cyclical oscillations in both economies.

Figure 3: Empirical Phase Diagrams

(a) Fixed Exchange Rate **(b)** Flexible Exchange Rate



Notes: The data is sourced from the IMF, World Bank, and the OECD. The primary fiscal surplus and deficit are illustrated by a positive and negative primary fiscal balance, respectively. The central bank's stock of foreign assets is transformed into months of import cover, where the minimum international recommendation is three months. Further, an increase in the nominal exchange rate indicates a depreciation relative to the USD.

The Case of a Pure Float: Expansionary Depreciation. The following Proposition outlines that a debt-targeting fiscal rule produces non-cyclical adjustments when a nominal depreciation is expansionary.

Proposition 2.4 (Hitting the Bullseye: Expansionary Depreciation)**.** *The Domar fiscal rule Ω_{DC} is akin to a stock-flow consistent fiscal rule when a nominal depreciation is expansionary.*

The basic idea is that an expansionary depreciation raises the growth rate and reduces the accumulation of public debt, which requires a primary deficit for debt sustainability. Also, the expansionary currency depreciation produces an external surplus and contracts private consumption as the currency depreciation lowers the wage share. Given the emerging trade surplus and stagnation of domestic demand, the market expects a rapid appreciation of the nominal exchange rate. Therefore, goods market equilibrium and exchange rate stability require a primary deficit. Note carefully that unlike Propositions 2.2 and 2.3, the fiscal response to stock-flow disequilibria is the same—a primary deficit. Hence, expansionary currency depreciation and non-cyclical adjustments.

Propositions 2.2 and 2.3 lead to the following Theorem about the government's budget identity and debt analysis.

Theorem 2.1 (Government's Budget Identity and Stock-Flow Inconsistency)**.** *Any debt sustainability condition derived from the government's budget identity is not stock-flow consistent, ignoring the case of expansionary depreciation.*

This Theorem indicates that a debt-targeting fiscal rule derived from the government's intertemporal budget constraint produces overshooting and undershooting of the stable debt trajectory as a matter of design and thereby; engenders cycles of debt crises.

3 Model

This section introduces the stock-flow model in continuous time. The basic setup starts with accounting identities to provide stock-flow consistency of income flows and financial stocks. After coherent stock-flow accounting, the model introduces behavioural formulations to analyse debt sustainability.

3.1 Environment

This sub-section introduces the key ingredients of the environment. The economy is small and open and consists of firms (bank and non-bank), rich and poor households, a government, and a central bank.

Households. The economy is populated by rich and poor households who maximise utility:

Definition 3.1 (Rich and Poor Households)**.** *Rich households are shareholders of commercial banks and non-financial corporations, while poor households are not.*

This Definition demonstrates the stylised fact that institutional investors—financial and non-financial corporations—are the principal holders of government securities on behalf of rich households.[11] Poor households are too poor to save in government securities and do not hold foreign-currency-denominated assets and liabilities. Instead, they purchase foreign currencies from commercial banks to engage in international transactions. These characteristics are summarised in the following Assumption and Balance Sheet.

Assumption 3.1 (Balance Sheet: Poor Households)**.** *Poor households do not hold government securities but have other assets and liabilities denominated in local currency units: bank deposits* $(D_{ph}) =$ *domestic debt* (B_{ph}).

Following Definition 3.1, the balance sheets of rich households are equivalent to the net worth of banks and firms.

Firms. Non-financial firms maximise profits and hold foreign-currency-denominated assets and liabilities to finance investment and engage in international trade. They also save by holding government securities. The following Assumption summarises these features, where c denotes corporation or firm.

Assumption 3.2 (Balance Sheet: Corporations)**.** *Firms' balance sheet consists of the following components: gov. debt* (B_g^c) + *foreign assets* (F_c) + *bank deposits* (D_c) = *foreign debt* (B_c^f) + *domestic debt* (B_c).

Banks. Financial intermediaries maximise profits but do not participate in carry-trade because interest rate differentials account for country-risk premium and do not provide profitable arbitrage opportunities. Also, banks satisfy the prudential regulations of required liquidity and non-remunerated reserves. Further, the banking system dominates the trade in foreign currency and holds government securities for liquidity and regulatory purposes through one of two channels:

1. Open market operations (OMOs) in the secondary market (the case of advanced economies and emerging markets), and

2. Open market type operations (OMTOs) in the primary market (the case of low-income countries).

The components of banks' balance sheets are specified below, where firms' and households' assets are liabilities to the banking system, and their liabilities are assets held by the banks.

[11] See footnote 6 for evidence.

Assumption 3.3 (Balance Sheet: Banks)**.** *Banks' balance sheet: high-powered money (H) +* B_g^b *+* $F_b + B_c + B_{ph} = D_c + D_{ph}$.

As banks extend credit to households and firms, they raise their target stock of high-powered money (H^T) to facilitate inter-bank clearing and the demand for cash. To maintain an effective payments system, the central bank accommodates this demand, which implies that the money supply is endogenous in the long run and exclusively determined by bank behaviour as follows:

$$\dot{H} = \delta(H^T - H), \tag{6a}$$

where $1 < \delta < 0$ is an adjustment parameter, and the stock of private sector debt $B_p = B_c + B_{ph}$.

$$H^T = \omega_0 + \omega_1 B_p \tag{6b}$$

The following Axiom summarises this insight.

Axiom 3.1 (Endogenous Money)**.** *In the long-run steady state, the extent of credit creation in the banking system determines the supply of high-powered money.*

Consolidated Public Sector: Central Bank and Government. The central government issues debt in local currency units to the private sector—rich households, firms, and banks—in the primary market, but foreign-currency-denominated debt in world markets. The latter point indicates that the Original Sin holds in this economy.

The monetary authority has one of two objectives depending on the exchange rate regime:

1. Fixed Peg: The central bank maintains a credible exchange rate peg $\bar{q}$, or
2. Pure Float: The central bank targets inflation.

In the case of a pure float, the uncovered interest rate parity condition adjusted for country risk anchors the central bank's policy rate. Moreover, the monetary authority undertakes OMOs with government securities purchased in the secondary market. In the case of OMTOs, the government issues the securities directly to the banking system. This is in contrast to many low-income countries, where the central bank issues sterilisation bonds, but this distinction disappears in the context of the consolidated public sector. Also, from the adding-up constraint, the public sector's balance sheet is a *residual*, given the balance sheets of households, firms, and banks.

The following Axiom outlines how money creation affects the nominal exchange rate (Frankel 1983; Branson 1977).

Axiom 3.2 (Money-Financed Fiscal Deficits and the Nominal Exchange Rate)**.** *In a flexible exchange rate regime, monetised-fiscal deficits, that is, the central bank's accumulation of government bonds in the primary market or reduction in government deposits at the central bank, increase the long-run rate of nominal depreciation.*

As the exchange rate depreciates, it increases the local currency burden of external debt and raises the real interest rate. So, to avoid these complications, the central bank commits to a no-monetisation policy as outlined below. This Assumption implies that the supply of high-powered money is endogenous (Axiom 3.1).

Assumption 3.4 (No-Monetisation Policy)**.** *The central bank does not monetise the fiscal deficit.*

In the case of a fixed peg, the central bank undertakes sterilised foreign exchange intervention with government securities to maintain a credible peg.

Assumption 3.5 (Complete Sterilisation)**.** *The accumulation of foreign assets and reductions in the government's deposit balance are completely sterilised through OMOs or OMTOs.*

This Assumption captures the basic premise of monetary policy in open economies with a credible peg, that is, intervention in the foreign exchange market and changes in the government's deposit balance should not affect the supply of high-powered money (Burnside et al. 2005; Frenkel and Johnson 1976). Otherwise, the market expects a currency devaluation, and the central bank loses control of its peg (Flood and Garber 1984; Krugman 1979).

The following Axiom states that monetised fiscal expenditures are operationalised through the stock of foreign assets held by the central bank.

Axiom 3.3 (Money-Financed Fiscal Deficits and Foreign Reserves)**.** *In a fixed exchange rate regime, monetised-fiscal deficits, that is, the central bank's accumulation of government bonds in the primary market or reduction in government deposits at the central bank, reduce the stock of foreign assets held by the central bank.*

The basic intuition is as follows. A competent central bank is keen to ensure that its purchase of government bonds in the primary market does not increase the supply of high-powered money to maintain a credible peg. Consequently, it adjusts its balance sheet by selling foreign currencies in the local market, which sterilises the accumulation of high-powered money. The key implication of this result is that the supply of high-powered money is also endogenous in the case of a fixed peg per Axiom 3.1.

To obtain a credible peg, the central bank maintains a stock of foreign assets consistent with the peg. Equation (7a) illustrates that the central bank accumulates foreign assets when its target stock of foreign reserves increases (F^T), where $1 < \gamma < 0$ is an adjustment parameter.

$$\dot{F}_{cb} = \gamma(F^T - F_{cb}) \tag{7a}$$

Equation (7b) specifies the determinants of the target stock of foreign assets held by the central bank, where $\alpha = B_g^f/B$ is the government's external debt as a share of total government debt and q^e is the expected nominal devaluation. Therefore, the central bank increases its demand for foreign assets when the government's external debt stock increases ($B\alpha = B_g^f$), which is necessary to service debt denominated in foreign currency. However, the central bank lowers its demand or target level of foreign assets when there is an expected devaluation ($q^e > 0$). Note that $q^e = 0$ when the market's expected exchange rate is consistent with the fixed peg, which is assumed to hold in the long run.

$$F^T = \rho_0 + \rho_1 B\alpha - \rho_2(q^e) \tag{7b}$$

Assumption 3.6 (A Credible Peg)**.** *The central bank maintains a credible peg:* $q^e = 0$.

Thus, the size of external debt determines the long-run stock of foreign assets.

3.2 Stock-Flow Equilibria: Fixed Peg

This sub-section introduces the stock-flow model for the case of a fixed peg, and $\bar{q}$ is omitted from the analysis.

Goods Market Flow Condition. Consider Condition (8a), which states that the goods market clears at full capacity ($\bar{Y}$). This point is summarised in the following assumption.

$$\bar{Y} = C + I + \tilde{G} + X - M \tag{8a}$$

Assumption 3.7 (Full Employment)**.** *In the long-run steady state, the economy obtains full em-*

ployment, where the output gap or excess capacity (beyond the normal level) is zero.

Given this assumption, Condition (8a) can be written as follows:

$$\tilde{G} + X + I = T + M + S, \tag{8b}$$

where $\tilde{G}$, X, and I are government outlays on goods/services and interest payments, exports, and investment, respectively, and T, M, and S are tax revenue, imports, and savings, respectively. Note that total government expenditure is $\tilde{G} = G + iB$, where i is the weighted nominal interest rate, and B is the sum of domestic and foreign public debt. Condition (8b) can be rewritten in terms of the government's primary fiscal balance as shown below:

$$G - T = M - X + S - I - iB, \tag{9}$$

which demonstrates that external deficits ($M - X$) or private sector savings ($S - I$) require a primary fiscal deficit to obtain goods market equilibrium at potential output, while higher interest payments require primary surpluses. In the latter case, public debt is a form of wealth or asset held by the private sector, and as iB increases, so does consumption and private demand. Thus, the primary deficit and public sector borrowing contracts to obtain full employment with stable prices. This mechanism ensures that public debt does not rise without limit.

Goods Market Stock-Flow Condition. Condition (10) identifies how trade deficits are financed, and a dot ($\cdot$) over a variable indicates its time derivative. Note that the subscripts g, cb, c, and b refer to the government, central bank, non-financial corporation and commercial bank, respectively, while the superscript f indicates a foreign currency-denominated asset or liability.

$$M - X = \overbrace{\dot{B}_g^f - \dot{F}_{cb}}^{\text{Public Sector}} + \overbrace{\dot{B}_c^f - \dot{F}_c - \dot{F}_b}^{\text{Rich Household}} \tag{10}$$

Import receipts exceed export earnings if the public sector reduces its net foreign assets by accumulating foreign currency debt ($\dot{B}_g^f$) or reducing its stock of foreign assets ($\dot{F}_{cb}$). In turn, imports may exceed exports if rich households reduce their net foreign assets, that is, when non-financial firms borrow in world markets ($\dot{B}_c^f$) or corporations ($\dot{F}_c$) and banks ($\dot{F}_b$) reduce their stocks of foreign assets. Banks reduce their holdings of foreign securities when poor households increase their purchase of foreign currencies to pay for imports.

Condition (11a) specifies the stock-flow dynamics of private sector saving, where the subscript *ph* indicates poor households.

$$S - I = \overbrace{\underbrace{\dot{H} + \dot{B}_g^b + \dot{B}_c + \dot{B}_{ph} + \dot{F}_b - \dot{D}_c - \dot{D}_{ph}}_{\text{Commercial Bank}} + \underbrace{\dot{B}_g^c + \dot{F}_c + \dot{D}_c - \dot{B}_c - \dot{B}_c^f}_{\text{Corporation}}}^{\text{Rich Household}} + \underbrace{\dot{B}_g^{ph} + \dot{D}_{ph} - \dot{B}_{ph}}_{\text{Poor Household}} \quad (11a)$$

A poor household saves by accumulating government debt ($\dot{B}_g^{ph}$) and bank deposits ($\dot{D}_{ph}$), and dissaves by borrowing from the banking system ($\dot{B}_{ph}$). Similarly, a non-financial corporation saves by the accumulation of government debt ($\dot{B}_g^c$), foreign assets, and bank deposits ($\dot{D}_c$); and dissaves by issuing domestic ($\dot{B}_c$) and external debt. Finally, the commercial bank builds its balance sheet by increasing its stock of high-powered money ($\dot{H}$), government securities ($\dot{B}_g^b$), corporate and poor household debt and foreign assets. However, the commercial bank's net wealth falls when the stock of bank deposits held by corporations and poor households increases.

Condition (11a) is simplified below, which demonstrates that rich households determine the private sector saving-investment balance:

$$S - I = \overbrace{\underbrace{\dot{H} + \dot{B}_g^b + \dot{F}_b}_{\text{Commercial Bank}} + \underbrace{\dot{B}_g^c + \dot{F}_c - \dot{B}_c^f}_{\text{Corporation}}}^{\text{Rich Household}} + \underbrace{\dot{B}_g^{ph}}_{\text{Poor Household}} = 0, \quad (11b)$$

where $\dot{B}_g^{ph} = 0$ per Assumption 3.1. Also, several studies have documented that poor households do not save in the form of government securities or have a negative net wealth balance.[12]

Substitution of Conditions (10) and (11b) into (9) yields the government's primary balance that is *stock-flow consistent*:

$$G - T = (\dot{B}_g^f + \dot{B}_g) + \dot{H} - \dot{F}_{cb} - iB, \quad (12)$$

where the omitted terms cancel out and $\dot{B}_g = \dot{B}_g^b + \dot{B}_g^c$. This result shows that the accumulation of foreign and domestic public debt and high-powered money fund the government's primary fiscal balance. In turn, the accumulation of foreign assets by the central bank ($\dot{F}_{cb}$) and an increase in

[12] See footnote 6 for evidence.

private sector wealth (iB) require a smaller primary deficit to stabilise the goods market at potential output with stable prices. The former relates to the rapid growth of the external sector, and the latter captures wealth-induced private consumption. This result is similar to the well-known government budget identity in Condition (1). However, the key difference is that the latter is not stock-flow consistent as it omits the external sector ($\dot{F}_{cb}$) and internal balance (endogenous high-powered money ($\dot{H}$)).

It is worth reminding the reader that the accumulation of high-powered money is exclusively demand-determined per Axiom 3.1, so it is *not a source* of government spending. Thus, the accumulation of high-powered money or bank loans increases the long-run saving rate on two counts: 1. The creditors' stock of wealth rises, and 2. The debtors increase their long-run savings to service their debt obligations. Consequently, the primary deficit increases to stabilise the goods market at full employment.

Stock-Flow Consistent Debt Dynamics. Recall that the total stock of public debt (B) is the sum of the government's foreign and domestic bond issues so the following holds.

$$\dot{B} = \dot{B}_g^F + \dot{B}_g$$

Then, Condition (12) is rewritten in terms of total public debt:

$$G - T = \dot{B} + \dot{H} - \dot{F}_{cb} - iB,$$

and rearranging this result in terms of $\dot{B}$ derives the evolution of total government debt that is stock-flow consistent.

$$\dot{B} = (G - T) - \dot{H} + \dot{F}_{cb} + iB \tag{13}$$

Recall Condition (2) that the stock of public debt is:

$$B = (b)PY,$$

and after the substitution of this result into Condition (13) yields:

$$\dot{B} = (G - T) - \dot{H} + \dot{F}_{cb} + i(b)P\bar{Y}. \tag{14}$$

Also, recall that the time derivative of the stock of public debt as a share of GDP is given by:

$$\frac{\dot{B}}{PY} = \frac{\dot{b}PY + \dot{P}bY + \dot{Y}bP}{PY},$$

and simplifies to the following.

$$\dot{b} = \frac{\dot{B}}{P\bar{Y}} + b(-\pi - g)b$$

Substitution of Condition (14) into this result yields:

$$\dot{b} = \frac{(G-T) - \dot{H} + \dot{F}_{cb}}{P\bar{Y}} + (i - \pi - g)b.$$

After invoking the Fisher equation ($r = i - \pi$), the dynamic evolution of the public debt ratio is given below:

$$\dot{b} = \frac{(G-T) - \dot{H} + \dot{F}_{cb}}{P\bar{Y}} + (r-g)b, \tag{15}$$

where r is the *weighted* real rate of interest. The weights are the respective shares of foreign ($\alpha = B_g^f/B$) and domestic debt ($1 - \alpha = B_g/B$) in total government debt.

Equation (15) indicates that the government's primary balance, the accumulation of high-powered money and foreign assets held by the central bank, and the difference between the weighted real interest rate and economic growth determine the dynamics of the public debt-GDP ratio.

The solution of the model depends on the dynamics of endogenous money and foreign assets held by the central bank. Substitution of Equations (6a-7b) into (15) solves the model and yields an augmented-Domar condition (ADC) that is stock-flow consistent, where the constants $\delta\omega_0$ and $\gamma\rho_0$ are omitted for simplicity.

$$\dot{b} = \Omega - \delta\omega_1 b_p + \delta h - \gamma f_{cb} + \Big[(r + \gamma\rho_1\alpha) - g\Big]b \tag{16}$$

The above result shows that the dynamics of total public debt as a share of GDP depend on the following: 1. The primary deficit as a share of GDP, 2. The stock of private sector debt as a share of GDP, 3. High-powered money as a share of GDP, 4. Foreign assets held by the central bank as a share of GDP, 5. External debt as a share of public debt, and 6. The weighted real interest rate-growth rate differential.

Intuition. (a) As private sector debt as a share of GDP increases, it decelerates the public debt ratio by reducing the long-run demand for government bonds. (i) When corporate debt rises, firms repay their liabilities by reducing their stocks of bank deposits, foreign assets, or government bonds. But corporate bank deposits are utilised for transaction purposes, while foreign assets service external corporate debt (recall Assumption 3.2). It follows that firms retire their stock of public debt to repay their domestic liabilities, which implies that their *demand constraint* for government bonds binds in the long run. Public debt is retired because foreigners do not hold local currency debt, and firms cannot sell government securities to rich households since they own the firms. (ii) The same channel is operational if firms or households repay their debt with bank deposits. This directly contracts their long-term demand for government bonds—the typical case in advanced economies and emerging markets. However, households or firms are not the prime holders of government securities in low-income economies. In this case, lower bank deposits produce excess bank reserves, and commercial banks purchase foreign assets from the central bank—instead of low-yielding government bonds above regulatory requirements (given the lower stock of deposits)—to maximise profitability and maintain a diversified portfolio. Such dishoarding of domestic assets means that banks demand fewer government bonds in the primary market. Overall, private sector debt repayment activates the demand constraint for government bonds.

(b). A higher stock of foreign assets held by the central bank as a share of GDP also actives the demand constraint. First, the government repays its external liabilities or incurs less external debt. Second, the private sector increases its stock of foreign assets and further contracts the long-term demand for government bonds. When the central bank's stock of foreign assets increases, households and firms can afford a larger volume of imports—consumption and intermediate capital goods—without compromising the exchange rate peg. To that end, households and firms retire public debt to accumulate foreign assets and purchase imported goods and services. (ii). The result is also the same if firms or households reduce their bank deposits to purchase foreign currencies from commercial banks. As explained earlier, a lower stock of deposits produces excess bank reserves and activates the demand constraint.

(c) An increase in the *endogenous* stock of high-powered money accelerates the public debt ratio because it raises voluntary savings in the form of government bonds. High-powered money reflects the rise in wealth that accrues to creditors (rich households) or shareholders of the banks. The transfer of assets from the debtor to the creditor increases savings that accumulate as government bonds. Note carefully that rich households may purchase foreign securities and undermine the peg if the government fails to issue more bonds in the primary market to satisfy their demand. But given Assumption 3.6 of a credible peg, the central government issues more bonds, and the debt ratio accelerates.

(d) A higher share of external debt accelerates the public debt ratio because more foreign assets service external debt obligations, which reduces the stock of foreign assets used for private consumption and investment. Then, the private sector accumulates public sector domestic debt as a form of forced savings.

3.3 Main Results: Fixed Peg

This sub-section presents the main results of the stock-flow model for the case of a fixed peg.

Recall that the Domar condition indicates that a long-run primary surplus stabilises the debt ratio when the economy is dynamically efficient: $r > g$ (Axiom 2.1). The following Theorem invalidates this claim.

Theorem 3.1 (Dynamic Efficiency and the Augmented-Domar Condition)**.** *When the economy is dynamically efficient:* $(r+\gamma\rho_1\alpha) > g$*, public debt is sustainable with a primary fiscal deficit as a share of GDP, if and only if:*

(a) the sum of private sector debt and foreign assets held by the central bank as shares of GDP exceeds the sum of high-powered money as a share of GDP and the weighted real interest rate-growth rate inequality: $-\delta\omega_1 b_p - \gamma f_{cb} > \delta h + (r+\gamma\rho_1\alpha - g)b$*.*

This result demonstrates that a long-run primary surplus is only feasible if the sum of high-powered money as a share of GDP and the weighted real interest rate-growth rate inequality is implausibly large.

There are several explanations for why a primary fiscal surplus is not consistent with debt sustainability. First, a long-run primary surplus implies that the private sector accumulates debt as a long-run outcome, which is not consistent with private-sector debt sustainability. (Leigh et al. 2012; Mian and Sufi 2010). The inevitable private sector debt crisis requires fiscal deficits for stabilisation purposes, which shows that the public debt dynamics and the fiscal surplus are unsustainable. Second, a long-run primary surplus requires a long-run current account surplus, which is only possible if trading partners accept long-run current account deficits. But even the USA finds it increasingly difficult to accept this long-run outcome. History and current events are also useful guides in this regard. They suggest that external imbalances adjust through various forms of conflict, for example, war, protectionism and financial crises.[13] It follows that a primary surplus does not provide for stable debt dynamics as it is a beggar-thy-neighbour policy with stringent political and economic limits—the case of Germany and the Eurozone crisis are recent examples (Storm and Naastepad 2015).

Third, a long-run primary surplus is not feasible in small and very-open economies. In this case, developmental requirements and political economy factors compel smaller primary surpluses or even deficits, which reduce the accumulation of foreign assets. Fourth, while exogenous shocks like natural disasters and global economic crises may provide justifications for primary surpluses, these are also not sustainable in the long run. Assuming that the primary surpluses are consistent with foreign asset accumulation and private sector savings (a reasonable assumption), then the long-run outcome is stagnation of domestic demand and an explosion of unemployment.[14] In other words, long-run fiscal and external surpluses with private sector savings necessarily contract the economy in the long run. It is not a stretch of the imagination to suggest that this is not a sustainable long-run outcome. Historical evidence, theory, and reason are not kind to the Theorem of a long-run primary surplus. Unlike the latter, primary deficits as a long-run outcome do not require adjustments. History and theory suggest that the corresponding private sector accumulation of assets can continue without limit unless interrupted by financial crises induced by speculation. But even these crises do not require primary surpluses as evidenced by the fiscal response of private sector bailouts. Moreover, as Theorem 3.1 underlines, a long-run primary deficit is consistent with a stable public debt ratio. On consideration of stock-flow equilibria, there is absolutely nothing odd or *irresponsible* about a long-run primary fiscal deficit.

The following Proposition specifies the speed of convergence to the long-run debt ratio.

Proposition 3.1 (Speed of Convergence)**.** *As compared to the Domar condition, the augmented-Domar condition (ADC) has a shorter half-life or a shorter period of convergence to its long-run steady state.*

The share of external debt in total public debt (α) drives this result as it reflects the risk of currency mismatch, which provides for an upper limit on external debt and faster convergence to the long-

[13] See Klein and Pettis (2020) for how global imbalances are underpinned by intra-country class conflict, and Delpeuch et al. (2021) for evidence that this leads to protectionism. The key point is that long-run primary surpluses become primary deficits to ameliorate intra-country class conflict.

[14] See Hein and Dodig (2015) for a discussion of this point using Germany as a recent example.

run debt ratio.

If there is nothing irresponsible about a long-run fiscal deficit, why do foreign exchange crises co-exist with fiscal deficits? The short answer is to blame the Domar fiscal rule Ω_{DC}, but the following Proposition provides further details.

Proposition 3.2 (Fiscal Rules and Foreign Exchange Crises)**.** *A fiscal rule that adheres to the Domar condition has a probability greater than zero of a foreign exchange crisis, while the augmented-Domar condition has a fiscal rule with a zero probability of foreign exchange crisis. Equation (17) presents the augmented-Domar fiscal rule (Ω_{ADC}) when $\dot{b} = 0$.*

$$\Omega_{ADC} = \delta \omega_1 b_p - \delta h + \gamma f_{cb} + (g - r - \gamma \rho_1 \alpha) b \tag{17}$$

This result shows that an open economy with a fixed exchange rate that obeys the augmented-Domar fiscal rule Ω_{ADC}—the stock-flow consistent fiscal rule—avoids foreign exchange crises and realises more stable economic performance.

Equation (17) shows that a primary deficit as a share of GDP is required to stabilise the public debt ratio when the long-run growth rate rises and the stock of private debt and foreign assets held by the central bank increases as a share of GDP. By now, the intuition is familiar. These factors induce a deceleration in the public debt ratio, and only a primary deficit stabilises debt. Conversely, fiscal austerity is required for debt stability as high-powered money rises as a share of GDP and the sum of the weighted real interest rate and share of external debt increases. In terms of goods market stability, a faster long-run growth rate expands productive capacity, and higher private-sector debt and foreign assets increase private savings and imports. These require a primary fiscal deficit for goods market equilibrium with stable prices. The reverse is true as high-powered money and interest income increase wealth-induced private demand. It follows that the long-run *fiscal space* is jointly determined by *resource availability* and *resource utilisation*. This point leads to the following Definition.

Definition 3.2 (Fiscal Space)**.** *Fiscal space refers to the extent of resource availability (foreign assets held by the central bank as a share of GDP) and the degree of resource utilisation (the long-run growth rate of potential output, private debt, high-powered money and interest income) consistent with full employment equilibrium and a stable debt ratio.*

This Definition outlines why the Domar fiscal rule Ω_{DC} misses the bullseye consistent with stock-flow equilibria (Propositions 2.2 and 2.3). The Domar fiscal rule violates the Definition of fiscal space and recommends a long-run primary fiscal surplus *irrespective* of the degree of resource utilisation. It is worth noting that fiscal space does not depend on the mobilisation of tax revenue denominated in local currency units in the fixed exchange rate open economy context. Also, an increase in the stock of foreign assets held by the central bank is *not* sufficient to expand the fiscal space. The latter also requires the under-utilisation of domestic resources (excess capacity). On this front, many open economies violate the Definition of fiscal space, thus, the cycle of foreign exchange crises.

The augmented-Domar condition is also instructive because it presents a compelling rationale for why open economies should accumulate foreign assets in the long run: to provide for stable public debt dynamics. The merit of the augmented-Domar condition is that it yields a country-specific and concrete guide to policymakers, unlike the arbitrary international recommendation that central banks hold foreign assets to cover three months of imports. The following Proposition summarises this insight.

Proposition 3.3 (Optimal Stock of Foreign Assets held by the Central Bank)**.** *The optimal stock of foreign assets as a share of GDP is given as follows:*

$$f_{cb}* = \frac{\Omega - \delta\omega_1 b_p + \delta h + (r + \rho_1\alpha - g)b}{\gamma}, \tag{18}$$

where $f_{cb}*$ is consistent with stable debt and goods market equilibrium at potential output.

Recall Axiom 3.3 that money-financed fiscal deficits lower the stock of foreign assets held by the central bank. Then, money creation explodes the public debt to GDP ratio and undermines debt sustainability. This result is summarised in the following Proposition.

Proposition 3.4 (Money Creation and Debt Sustainability)**.** *(a). Given Assumption 3.5 of complete sterilisation, money-financed fiscal deficits undermine the government's ability to service external debt as it lowers the stock of foreign assets held by the central bank, which compromises overall debt sustainability.*

(b). When Assumption 3.5 is relaxed, and there is incomplete sterilisation, that is, growth in high-powered money due to monetisation, debt becomes unsustainable through two channels: (i). Difficulty in external debt service due to a reduced stock of foreign assets held by the central bank, and (ii). A higher stock of high-powered money expands bank deposits and raises the demand for

government bonds.

Overall, monetisation undermines debt sustainability unless the inflationary effect is implausibly large. Closed-economy models, which are stock-flow inconsistent, show that monetisation reduces the real debt burden (Buiter et al. 1985). However, the latter is not empirically relevant in the open economy with a fixed peg, where monetisation reduces the stock of foreign assets and undermines external debt service and the credibility of the peg. Unlike the closed-economy view of monetisation, these adverse effects are more likely to increase the weighted real interest rate than reduce it.

Remark 3.1 (Remittances and Trade-Weighted Exchange Rate)**.** Due to brevity and space, the stock-flow debt model omits two empirically relevant factors. First, remittances as a share of GDP provide for more stable debt dynamics as it increases the stock of foreign assets. Second, even a fixed peg is subject to trade-weighted exchange rate risks, which can accelerate the public debt ratio depending on the currency composition of external debt (Burnside 2005). Also, these factors affect the augmented-Domar fiscal rule and the optimal stock of foreign assets held by the central bank.

3.4 Stock-Flow Equilibria: Pure Float

This sub-section introduces a flexible exchange rate regime into the stock-flow model.

The stock-flow consistent primary balance for the case of a pure float is as follows:

$$G-T=(q\dot{B}_g^f+\dot{B}_g)+\dot{H}-qiB, \tag{19a}$$

where $\dot{F}_{cb}=0$ in a flexible exchange rate regime, and an increase in q denotes a nominal depreciation.

The government's total public debt is the sum of its domestic and foreign bond issues, so the following holds.

$$q\dot{B}=q\dot{B}_g^f+\dot{B}_g$$

Thus, Condition (19a) is rewritten as:

$$G-T=q\dot{B}+\dot{H}-qiB, \tag{19b}$$

and rearranging this result in terms of $q\dot{B}$ derives the evolution of total government debt.

$$q\dot{B} = (G-T) - \dot{H} + qiB \tag{20}$$

Recall Condition (2) that the stock of public debt is defined as:

$$qB = (b)PY.$$

and after the substitution of this result into Condition (20) yields:

$$q\dot{B} = (G-T) - \dot{H} + i(b)PY. \tag{21}$$

Also, recall that the time derivative of the stock of public debt as a share of GDP is given by:

$$\frac{\dot{q}B}{PY} + \frac{q\dot{B}}{PY} = \frac{\dot{b}PY + \dot{P}bY + \dot{Y}bP}{PY},$$

which is simplified to the following, where $g_q = \dot{q}/q$ is the long-run rate of nominal depreciation.

$$\dot{b} = \frac{q\dot{B}}{PY} + (g_q - \pi - g)b$$

Substitution of Condition (21) into this result yields:

$$\dot{b} = \frac{(G-T) - \dot{H}}{PY} + (i + g_q - \pi - g)b.$$

After invoking the augmented Fisher equation $(r' = i + g_q - \pi)$, the dynamic evolution of public debt ratio is given below:

$$\dot{b} = \frac{(G-T) - \dot{H}}{PY} + (r' - g)b, \tag{22}$$

where r' is the weighted real interest rate and accounts for the long-run rate of nominal depreciation. The weights are the respective shares of foreign and domestic debt in total government debt.

Substitution of Equation (6a) and (6b) into (22) solves the stock-flow model, where the constant

$\delta\omega_0$ is omitted for convenience.

$$\dot{b} = \Omega - \delta\omega_1 b_p + \delta h + (r' - g)b. \tag{23}$$

Intuition. This result is the same as the case of a fixed peg with one exception: a nominal depreciation increases the weighted real interest rate and accelerates the public debt ratio because it raises the local currency burden of external debt.

3.5 Main Results: Pure Float

This sub-section presents the main results of the stock-flow model for the case of a pure float.

The following Theorem outlines the condition for debt sustainability under the case of dynamic efficiency.

Theorem 3.2 (Dynamic Efficiency and Debt Sustainability)**.** *When the economy is dynamically efficient: $r' > g$, debt becomes sustainable with a primary fiscal deficit as a share of GDP, if and only if:*

(a) the private sector's domestic debt as a share of GDP exceeds the sum of high-powered money as a share of GDP and the weighted interest rate-growth rate inequality: $-\delta\omega_1 b_p > \delta h + (r' - g)b$.

This result parallels Theorem 3.1 in terms of foreign exchange risk: the share of external debt in total government debt (α) reflects this risk in the case of a fixed peg, while the long-run rate of nominal depreciation that increases r' serves this function in the case of a pure float. However, it is transparent that this result is weaker when compared to a fixed peg as it omits foreign assets held by the central. It follows that flexible exchange rate regimes require smaller fiscal deficits to stabilise debt and the goods market. Comparing the Y-axes in Figure 3 verifies this insight. It is also intuitive as exchange rate depreciation limits the size of fiscal deficits on consideration of debt and goods market stability.

The following Proposition specifies the speed of convergence to the long-run debt ratio.

Proposition 3.5 (Speed of Convergence)**.** *As compared to the Domar condition, the augmented-Domar condition has a shorter half-life or period of convergence to its long-run steady state.*

The long-run rate of nominal depreciation drives this result, which establishes an upper limit on the size of external debt and provides for faster convergence to the long-run debt ratio.

Equation (24) presents the augmented-Domar fiscal rule (Ω^{fxr}_{ADC}) when $\dot{b} = 0$, where the superscript fxr denotes the flexible exchange rate regime.

$$\Omega^{fxr}_{ADC} = \delta \omega_1 b_p - \delta h + (g - r')b \tag{24}$$

The fiscal rule Ω^{fxr}_{ADC} is stock-flow consistent because it accounts for stable debt dynamics and exchange rate stability consistent with full employment. The following Proposition summarises this point.

Proposition 3.6 (SFC Fiscal Rule and Exchange Rate Stability)**.** *A fiscal rule that adheres to the augmented-Domar condition Ω^{fxr}_{ADC} provides for both debt and exchange rate stability.*

This result demonstrates that exchange rate volatility is the price paid for following a stock-flow inconsistent fiscal rule (recall Figure 3(b)). In other words, a debt-targeting fiscal rule produces enormous swings in the nominal exchange rate as a matter of design, which has feedback effects on the dynamics of public debt. Volatile exchange rates justify fixed or managed exchange rate regimes, but this result shows that the stock-flow consistent fiscal rule Ω^{fxr}_{ADC} is sufficient.

The following Proposition outlines how money creation affects the dynamics of public debt.

Proposition 3.7 (Money Creation and Debt Sustainability)**.** *When Assumption 3.4 is relaxed, the central bank monetises the fiscal deficit and undermines debt sustainability through two channels: (i). It raises the stock of high-powered money as a share of GDP and (ii). It increases the long-run rate of nominal depreciation following Axiom 3.2.*

Recall that the weighted real interest rate is $i + g_q - \pi$, so it is straightforward that monetisation accelerates the public debt ratio as the long-run rate of nominal depreciation increases (g_q). But a nominal depreciation is also inflationary in open economies through the import channel and may reduce the real debt burden. However, it also reduces real wages and may induce a wage-price spiral, and domestic bondholders may demand a higher real interest rate to protect their domestic purchasing power. Also, external creditors may impose a higher real interest rate to account for exchange rate risks. In short, unlike the standard claim of inflating away the debt, monetisation is more likely to increase the real debt burden in open economies with a pure float.

4 Conclusion

Debt sustainability analyses are fundamentally forward-looking exercises that involve forecasts and economic judgments about future interest and growth rates, nominal exchange rate, current account, and primary balances. Different combinations of these forecast variables inform domestic and external debt sustainability analyses, where stable debt refers to a steady-state debt ratio relative to some threshold. The standard debt sustainability condition emerges from the government's intertemporal budget constraint and omits goods market (flow) equilibrium. Consequently, the debt-targeting fiscal rule overshoots/undershoots the steady-state debt ratio. In other words, the standard debt sustainability analysis is not stock-flow consistent, which is necessary for appropriate analysis as debt ratios combine stock (debt) and flow (GDP or exports) variables. This research shows that the price of stock-flow inconsistency is significant volatility: debt or foreign exchange crises.

This article formalises a stock-flow consistent model of fiscal and debt sustainability in an open economy. It demonstrates that a primary deficit as a share of GDP obtains goods market equilibrium at potential output and a steady-state debt ratio, irrespective of the exchange rate regime and even when the economy is dynamically efficient. The model derives a simple rule that specifies the precise primary deficit required for stock-flow equilibria.

It is worthwhile to reconcile this result with historical evidence. In excellent historical research on public debt, Eichengreen et al. (2019) documents the successful fiscal consolidation episodes in Great Britain, France, and the USA before 1913. Two points are in order. First, before 1913, debt instruments were not fundamental to the operation of monetary policy, where the central bank influences the yield curve and provides for a long-run fiscal deficit. Second, deficit financing as a means to stabilise the goods market at full employment was not part of the economic orthodoxy until several years after the publication of The General Theory (Keynes 1936). Thus, this historical research is not particularly useful to modern-day policy. In related work, Mauro et al. (2015) employ a historical dataset that covers 55 countries for up to two hundred years and find evidence that the response of the primary fiscal surplus to variation in government debt is consistent with meeting governments' intertemporal budget constraint for the period before 2008. They find evidence of primary surpluses in most advanced economies during the mid-1990s and in emerging economies after 2000. Stock-flow equilibria require that these primary surpluses accelerate private sector indebtedness, which has occurred and underpinned the recent global financial crisis (GFC)

(Leigh et al. 2012; Mian and Sufi 2010). In my view, this best demonstrates that a long-run primary surplus is not sustainable as the economic consequences of the GFC require a primary deficit for stabilisation purposes.

There are three areas for future research. First, modeling the micro-foundations of the stock-flow equilibria is an area for further investigation. Second, possible extensions include contingent liabilities and floating interest rates on domestic and external debt. Third, this work presents new opportunities for empirical studies on stock-flow consistent fiscal and debt sustainability analyses.

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Appendix: Omitted Proofs

Proof of Proposition 2.1 This follows logically from Propositions 2.2 and 2.3. □

Proof of Proposition 2.2 This proof proceeds in two steps: 1. It derives the dynamic system that specifies the evolution of the government's primary fiscal balance and the stock of foreign assets held by the central bank, and 2. It evaluates the stability of the system and the properties of its dynamic adjustments.

Step 1. Consider the following dynamic adjustment of the primary fiscal balance, where Ω^T is the government's target primary balance and $0 < \mu < 1$ is an adjustment parameter.

$$\dot{\Omega} = \mu(\Omega^T - \Omega) \tag{A.1}$$

The government's target primary balance is determined by its debt targeting fiscal rule to obtain $\dot{b} = 0$.

$$\Omega^T \equiv \dot{b} = 0 \tag{A.2}$$

The Equation below reproduces the dynamic adjustment of the public debt ratio:

$$\dot{b} = \Omega + (r - g)b, \tag{A.3}$$

and Equation (A.4) shows that the long-run growth rate is a positive function of the central bank's stock of foreign assets (F_{cb}). The basic intuition is as follows. Long-run growth is determined by innovation and technological change, which rely on imported capital and the central bank's stock of foreign assets.

$$g = \theta_0 + \theta_1 F_{cb} \tag{A.4}$$

Substitution of Equation (A.4) into (A.3) derives the primary fiscal balance that is consistent with a constant public debt ratio when $\dot{b} = 0$. Note that this result is the government's target primary balance.

$$\Omega^T \equiv \Omega = b(\theta_0 + \theta_1 F_{cb} - r) \tag{A.5}$$

Thus, $\dot{\Omega}$ is given as follows:

$$\dot{\Omega} = \mu\Big(b(\theta_0 + \theta_1 F_{cb} - r) - \Omega\Big). \tag{A.6}$$

In turn, Equation (A.7) shows the evolution of the stocks of foreign assets held by the central bank, where g_{AD} is the growth rate of aggregate demand, and $0 < \nu < 1$ is an adjustment parameter. When the goods market clears, $g = g_{AD}$, and deficient and excess demand are given by $g > g_{AD}$ and $g < g_{AD}$, respectively.

$$\dot{F}_{cb} = \nu(g - g_{AD}) \tag{A.7}$$

To derive the aggregate demand growth rate, consider the standard macroeconomic relationship:

$$1 = c + i + \Omega + nx, \tag{A.8}$$

where c, i, and nx are consumption, investment, and net exports as shares of GDP respectively. The primary balance is given, and c, i, and nx are specified below. Note that these are simplistic specifications but sufficient for the task at hand. Each component of demand as a share of GDP is negatively related to the aggregate demand growth rate and positively related to the central bank's stock of foreign assets. The principal channel for the latter relates to the confidence that the pegged rate is well anchored. Note that growth in external demand z increases net exports as a share of GDP.

$$c = \psi_0 - \psi_1 g_{AD} + \psi_2 F_{cb} \tag{A.9}$$

$$i = \phi_0 - \phi_1 g_{AD} + \phi_2 F_{cb} \tag{A.10}$$

$$nx = \omega_0 + \omega_1 z - \omega_2 g_{AD} + \omega_3 F_{cb} \tag{A.11}$$

Substitution of Equations (A.9)-(A.11) into (A.8) derives the aggregate demand growth rate.

$$g_{AD} = \frac{\psi_0 + \phi_0 + \omega_0 - 1 + \omega_1 z + \Omega + F_{cb}(\psi_2 + \phi_2 + \omega_3)}{\psi_1 + \phi_1 + \omega_2} \tag{A.12}$$

In turn, the substitution of Equations (A.4) and (A.12) into (A.7) yields the dynamic adjustment of the stocks of foreign assets held by the central bank.

$$\dot{F}_{cb} = \nu\Big(\theta_0 + \theta_1 F_{cb} - \Big[\frac{\psi_0 + \phi_0 + \omega_0 - 1 + \omega_1 z + \Omega + F_{cb}(\psi_2 + \phi_2 + \omega_3)}{\psi_1 + \phi_1 + \omega_2}\Big]\Big) \tag{A.13}$$

Note that dynamic stability requires that $\theta_1 < \frac{-\psi_2 - \phi_2 - \omega_3}{\psi_1 + \phi_1 + \omega_2}$, otherwise, the central bank's stock of foreign assets explodes indefinitely, which is implausible. The basic intuition for this necessary condition is that as the central bank increases its stock of foreign assets, it improves confidence, which accelerates consumption and investment faster than the pace of foreign exchange-induced innovation.

The dynamic system is reproduced below.

$$\dot{\Omega} = \mu\Big(b(\theta_0 + \theta_1 F_{cb} - r) - \Omega\Big)$$

$$\dot{F}_{cb} = \nu\Big(\theta_0 + \theta_1 F_{cb} - \frac{\psi_0 + \phi_0 + \omega_0 - 1 + \omega_1 z + \Omega + F_{cb}(\psi_2 + \phi_2 + \omega_3)}{\psi_1 + \phi_1 + \omega_2}\Big)$$

Step 2. This system is evaluated by the terms of the Jacobian matrix.

$$J = \begin{bmatrix} \frac{d\dot{\Omega}}{d\Omega} & \frac{d\dot{\Omega}}{dF_{cb}} \\ \\ \frac{d\dot{F}_{cb}}{d\Omega} & \frac{d\dot{F}_{cb}}{dF_{cb}} \end{bmatrix} = \begin{bmatrix} -1 & b\theta_1 \\ \\ \frac{-1}{\psi_1+\phi_1+\omega_2} & \frac{-\psi_2-\phi_2-\omega_3}{\psi_1+\phi_1+\omega_2} + \theta_1 \end{bmatrix}$$

The trace of the Jacobian matrix is $-1 + \frac{-\psi_2-\phi_2-\omega_3}{\psi_1+\phi_1+\omega_2} + \theta_1 < 0$, and hence always negative—recall that $\theta_1 < \frac{\psi_2+\phi_2+\omega_3}{\psi_1+\phi_1+\omega_2}$. However, the determinant of the system is ambiguous as $\frac{\psi_2+\phi_2+\omega_3}{\psi_1+\phi_1+\omega_2} + \theta_1 - (b\theta_1)\Big(\frac{1}{\psi_1+\phi_1+\omega_2}\Big) \lesseqgtr 0$. The system is dynamically stable if the determinant is positive, otherwise, a saddle-point (unstable) equilibrium exists. Since the product of the off-diagonal elements of the matrix is negative $(b\theta_1)\Big(\frac{-1}{\psi_1+\phi_1+\omega_2}\Big) < 0$, the system satisfies the necessary condition for oscillations or cyclical dynamics. Thus, a saddle-point can be ruled out, which implies that the system has a stable focus, consistent with the cyclical adjustments in Figure 1.

The positively sloped locus $\dot{b} = 0$ in Figure 1 is given by:

$$\Omega_{b*} = b(z + z_1 F_{cb} - r),$$

where an increase in F_{cb} accelerates long-run growth and contracts the public debt ratio, which requires a bigger fiscal deficit to stabilise debt. In turn, the negatively sloped locus $\dot{F}_{cb} = 0$ in Figure 1 is shown below, where $\psi_2 + \phi_2 + \omega_3 > (\theta_1)(\psi_1 + \phi_1 + \omega_2)$.

$$\Omega_{fcb*} = (\psi_1 + \phi_1 + \omega_2)\theta_0 - \omega_1 z + 1 - \psi_0 - \phi_0 - \omega_0 - F_{cb}\Big(\psi_2 + \phi_2 + \omega_3 - (\theta_1)(\psi_1 + \phi_1 + \omega_2)\Big)$$

This result shows that as F_{cb} increases, it generates excess demand in the goods market, so fiscal contraction is necessary for goods market equilibrium.

It is worth highlighting that though the system is dynamically stable, there are several periods of boom and bust, and sufficiently deep downturns stabilise debt and foreign assets held by the central bank. □

Proof of Proposition 2.3 This proof proceeds in two steps: 1. It derives the dynamic system that specifies the evolution of the government's primary fiscal balance and the nominal exchange rate when a nominal depreciation is contractionary, and 2. It evaluates the stability of the system and the properties of its dynamic adjustments.

Step 1. Equations (A.14) and (A.15) show the cases of contractionary and expansionary depreciation, respectively. There are several channels at work in the contractionary case. First, as a nominal depreciation depresses the economy, innovation and technological progress are undermined due to weaker learning-by-doing effects. Second, the currency depreciation and the depressed economy lower the firm-level wage share to such an extent that there are disincentives to invest in labour-saving technology. Finally, a currency depreciation increases the costs of imported technology/capital in local currency units and the cost of external finance (loan rate), both of which crowd out firm-level innovation expenditure. The long-run growth rate decreases as a consequence. These effects are reversed in the expansionary case so that a nominal depreciation increases long-run growth. In this case, the learning-by-doing effects and the lower firm-level wage share are sufficiently strong to induce technological progress and increase long-run growth.

$$g = \eta_0 - \eta_1 q, \text{ contractionary case} \tag{A.14}$$

$$g = \eta_0 + \eta_1 q, \text{ expansionary case} \tag{A.15}$$

Given Equations (A.1-A.3), the government's primary fiscal balance evolves as follows.

$$\dot{\Omega} = \mu\Big(b(\eta_0 - \eta_1 q - r) - \Omega\Big), \text{ contractionary case} \tag{A.16}$$

$$\dot{\Omega} = \mu\Big(b(\eta_0 + \eta_1 q - r) - \Omega\Big), \text{ expansionary case} \tag{A.17}$$

The nominal exchange rate evolves according to the following dynamic specification, where $0 < \lambda < 1$ is an adjustment parameter, and q^e is the expected exchange rate. When the latter increases, this accelerates the rate of depreciation until the long-run rate adjusts to the market's expectation.

$$\dot{q} = \lambda(q^e - q) \tag{A.18}$$

When the goods market clears ($g_{AD} = g$), the market's expectation is consistent with the long-run trend. However, the cases of deficient ($g_{AD} < g$) and excess demand ($g_{AD} > g$) appreciate and depreciate the nominal exchange rate, respectively. This is summarised below.

$$q^e = \kappa_0 + \kappa_1(g_{AD} - g) \tag{A.19}$$

Next, I formulate the growth rate of aggregate demand in the contractionary and expansionary cases. Recall that c, i, and nx are specified as follows with the inclusion of the nominal exchange. It is transparent that the latter has contractionary effects on c and nx but increases i. Note that when a nominal currency depreciation is contractionary, $\psi_2 + \omega_3 > \phi_2$.

$$c = \psi_0 - \psi_1 g_{AD} - \psi_2 q \tag{A.20a}$$

$$i = \phi_0 - \phi_1 g_{AD} + \phi_2 q \tag{A.20b}$$

$$nx = \omega_0 + \omega_1 z - \omega_2 g_{AD} - \omega_3 q \tag{A.20c}$$

The following outlines the case of expansionary depreciation, where $\omega_3 + \phi_2 > \psi_2$. An expansionary currency depreciation increases net exports and investment demand relative to consumption.

$$c = \psi_0 - \psi_1 g_{AD} - \psi_2 q \tag{A.21a}$$

$$i = \phi_0 - \phi_1 g_{AD} + \phi_2 q \tag{A.21b}$$

$$nx = \omega_0 + \omega_1 z - \omega_2 g_{AD} + \omega_3 q \tag{A.21c}$$

Substitution of Equations (A.20a-A.20c) into (A.8) derives the aggregate demand growth rate in the contractionary case.

$$g_{AD} = \frac{\psi_0 + \phi_0 + \omega_0 - 1 + \omega_1 z + \Omega - q(\psi_2 + \omega_3 - \phi_2)}{\psi_1 + \phi_1 + \omega_2}, \text{ contractionary case} \tag{A.22}$$

In turn, substitution of Equations (A.21a-A.21c) into (A.8) derives the aggregate demand growth rate in the expansionary case.

$$g_{AD} = \frac{\psi_0 + \phi_0 + \omega_0 - 1 + \omega_1 z + \Omega + q(\phi_2 + \omega_3 - \psi_2)}{\psi_1 + \phi_1 + \omega_2}, \text{ expansionary case} \tag{A.23}$$

Next, I formulate the dynamic adjustment of the nominal exchange rate. Substitution of Equations (A.14) and (A.22) into (A.19) and (A.18) yields the following.

$$\dot{q} = \lambda\Big(\kappa_0 + \kappa_1\Big[\frac{\psi_0 + \phi_0 + \omega_0 - 1 + \omega_1 z + \Omega - q(\psi_2 + \omega_3 - \phi_2)}{\psi_1 + \phi_1 + \omega_2} - \eta_0 + \eta_1 q\Big] - q\Big), \text{ contractionary case} \tag{A.24}$$

The dynamic system for the contractionary case (Figure 2) is reproduced below.

$$\dot{\Omega} = \mu\Big(b(\eta_0 - \eta_1 q - r) - \Omega\Big), \text{ contractionary case}$$

$$\dot{q} = \lambda\Big(\kappa_0 + \kappa_1\Big[\frac{\psi_0 + \phi_0 + \omega_0 - 1 + \omega_1 z + \Omega - q(\psi_2 + \omega_3 - \phi_2)}{\psi_1 + \phi_1 + \omega_2} - \eta_0 + \eta_1 q\Big] - q\Big), \text{ contractionary case}$$

Step 2. This system is evaluated by the terms of the Jacobian matrix.

$$J = \begin{bmatrix} \frac{d\dot{\Omega}}{d\Omega} & \frac{d\dot{\Omega}}{dq} \\ \\ \frac{d\dot{q}}{d\Omega} & \frac{d\dot{q}}{dq} \end{bmatrix} = \begin{bmatrix} -1 & -\eta_1 \\ \\ \frac{1}{\psi_1+\phi_1+\omega_2} & \frac{-\psi_2-\omega_3+\phi_2}{\psi_1+\phi_1+\omega_2} + \eta_1 - 1 \end{bmatrix}$$

The trace of the Jacobian matrix is always negative as $-1 + \frac{-\psi_2-\omega_3+\phi_2}{\psi_1+\phi_1+\omega_2} + \eta_1 - 1 < 0$. However, the determinant of the system is ambiguous as $\frac{\psi_2+\omega_3+\phi_2}{\psi_1+\phi_1+\omega_2}+\eta_1-1-(\eta_1)\left(\frac{1}{\psi_1+\phi_1+\omega_2}\right)\lesseqgtr 0$. Since the product of the off-diagonal elements of the matrix is negative $(-\eta_1)\left(\frac{1}{\psi_1+\phi_1+\omega_2}\right)<0$, the system satisfies the necessary condition for oscillations or cyclical dynamics. Thus, a saddle-point can be ruled out, which implies that the system has a stable focus, consistent with the cyclical adjustments in Figure 2.

The positively sloped locus $\dot{q}=0$ in Figure 2 is given by:

$$\Omega_q* = (\psi_1+\phi_1+\omega_2)(q-\eta_1 q+\eta_0-\kappa_0)+q(\psi_2+\omega_3-\phi_2)-\psi_0-\phi_0-\omega_0+1-\omega_1 z,$$

where an increase in q lowers the growth of aggregate demand and requires a fiscal deficit to stabilise the goods market. Recall that $\psi_2+\omega_3>\phi_2$ and it is transparent that $q>\eta_1 q$. In turn, the negatively sloped locus $\dot{b}=0$ in Figure 2 is shown below, where an increase in q accelerates the public debt ratio and requires a fiscal contraction for debt sustainability.

$$\Omega_{b*} = b(\eta_0-\eta_1 q-r)$$

□

Proof of Proposition 2.4 This proof proceeds in two steps: 1. It derives the dynamic system that specifies the evolution of the government's primary fiscal balance and the nominal exchange rate when a nominal depreciation is expansionary, and 2. It evaluates the stability of the system and the properties of its dynamic adjustments.

Step 1. Substitution of Equations (A.15) and (A.23) into (A.19) and (A.18) yields the dynamic adjustment of the nominal exchange rate.

$$\dot{q}=\lambda\left(\kappa_0+\kappa_1\left[\frac{\psi_0+\phi_0+\omega_0-1+\omega_1 z+\Omega+q(\phi_2+\omega_3-\psi_2)}{\psi_1+\phi_1+\omega_2}-\eta_0-\eta_1 q\right]-q\right), \text{ expansionary case} \tag{A.25}$$

The dynamic system for the expansionary case is reproduced below.

$$\dot{\Omega}=\mu\Big(b(\eta_0+\eta_1 q-r)-\Omega\Big), \text{ expansionary case}$$

$$\dot{q}=\lambda\left(\kappa_0+\kappa_1\left[\frac{\psi_0+\phi_0+\omega_0-1+\omega_1 z+\Omega+q(\phi_2+\omega_3-\psi_2)}{\psi_1+\phi_1+\omega_2}-\eta_0-\eta_1 q\right]-q\right), \text{ expansionary case}$$

Step 2. This system is evaluated by the terms of the Jacobian matrix.

$$J = \begin{bmatrix} \frac{d\dot{\Omega}}{d\Omega} & \frac{d\dot{\Omega}}{dq} \\ \\ \frac{d\dot{q}}{d\Omega} & \frac{d\dot{q}}{dq} \end{bmatrix} = \begin{bmatrix} -1 & \eta_1 \\ \\ \frac{1}{\psi_1+\phi_1+\omega_2} & \frac{\phi_2+\omega_3-\psi_2}{\psi_1+\phi_1+\omega_2} - \eta_1 - 1 \end{bmatrix}$$

The trace of the Jacobian matrix is always negative as $-1 + \frac{-\psi_2-\omega_3+\phi_2}{\psi_1+\phi_1+\omega_2} + \eta_1 - 1 < 0$. However, the determinant of the system is ambiguous as $\frac{\phi_2+\omega_3-\psi_2}{\psi_1+\phi_1+\omega_2} - \eta_1 - 1 - (\eta_1)\left(\frac{1}{\psi_1+\phi_1+\omega_2}\right) \lesseqgtr 0$. Since the product of the off-diagonal elements of the matrix is unambiguously positive $(\eta_1)\left(\frac{1}{\psi_1+\phi_1+\omega_2}\right) > 0$, the system does not satisfy the necessary condition for oscillations or cyclical dynamics. Thus, one of two equilibria is possible: 1. A saddle-point (unstable) equilibrium if the determinant is negative or, 2. A stable node if the determinant is positive. Figure 4 illustrates both possibilities, where a stable equilibrium (stable node) is only possible if the $\dot{b} = 0$ locus is flatter than the $\dot{q} = 0$ locus, otherwise, a saddle-point equilibrium is realised.

The positively sloped locus $\dot{q} = 0$ in Figure 4 is given by:

$$\Omega_q* = (\psi_1 + \phi_1 + \omega_2)(q + \eta_1 q + \eta_0 - \kappa_0) - q(\phi_2 + \omega_3 - \psi_2) - \psi_0 - \phi_0 - \omega_0 + 1 - \omega_1 z,$$

where $q + \eta_1 q > -q(\phi_2 + \omega_3 - \psi_2)$. Ergo, an expansionary depreciation requires a fiscal deficit to stabilise the goods market and the nominal exchange rate, given the market's expectation of a currency appreciation. In turn, the positively sloped locus $\dot{b} = 0$ is shown below, where an expansionary depreciation accelerates growth and reduces the public debt ratio, which requires a fiscal deficit for debt sustainability.

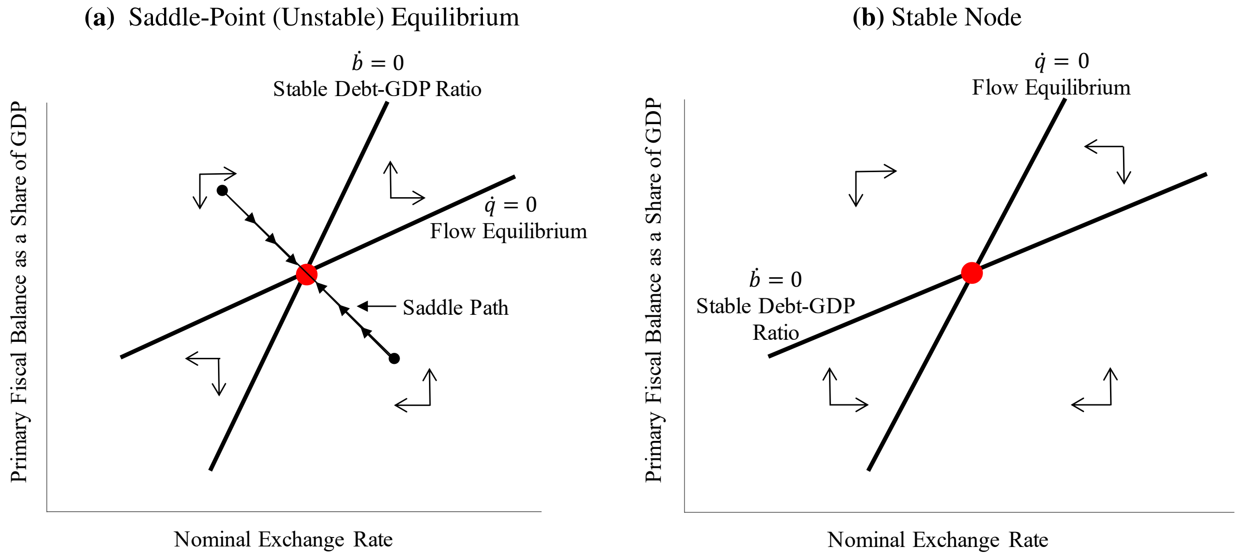
$$\Omega_{b*} = b(\eta_0 + \eta_1 q - r)$$

Proof of Theorem 2.1 Consider the government's budget identity expressed as a share of GDP.

$$\frac{G-T}{P\bar{Y}} = \frac{\dot{B}}{PY} + \frac{\dot{H}}{PY} - \frac{iB}{PY}$$

Since the standard approach assumes that $\dot{H} = 0$, the identity reduces to the following, which omits

Figure 4: The Case of an Expansionary Currency Depreciation



□

the external and internal balances.

$$\frac{G-T}{PY} = \frac{\dot{B}}{PY} - \frac{iB}{PY}$$

Recall that $\frac{\dot{B}}{PY}$ is given by

$$\frac{\dot{B}}{PY} = \frac{\dot{b}PY + \dot{P}bY + \dot{Y}bP}{PY},$$

and after substitution into the government's budget identity, the stock-flow inconsistent Domar condition is derived below.

$$\dot{b} = \Omega + (r-g)b$$

□

Proof of Theorem 3.1 This follows logically from Equation (16). □

Proof of Proposition 3.1 This proof is straightforward and is shown below.

$$\overbrace{\frac{ln(2)}{\Big((r+\gamma\rho_1\alpha)-g\Big)}}^{\text{Augmented-Domar Condition}} < \overbrace{\frac{ln(2)}{(r-g)}}^{\text{Domar Condition}}$$

□

Proof of Proposition 3.2 Following Proposition 2.1, the Domar condition is not stock-flow consistent, so its fiscal rule Ω_{DC} engenders swings of deficient and excess demand, where the latter is consistent with primary deficits, losses of foreign assets, and foreign exchange crises. Therefore, the Domar fiscal rule has a probability greater than zero that a foreign exchange crisis emerges, see Proposition 2.2. However, the augmented-Domar fiscal rule Ω_{ADC} is stock-flow consistent as it accounts for the private sector and external balances. It explicitly considers the risk of a currency mismatch by accounting for the stock of foreign assets as a share of GDP and the share of external debt in total public debt. Further, Ω_{ADC} avoids excess and deficient demand in the goods market by accounting for the private sector's domestic liabilities as a share of GDP. It follows that Ω_{ADC} has a probability of zero that a foreign exchange crisis emerges. □

Proof of Proposition 3.3 This follows from Equation (18) when $\dot{b} = 0$. □

Proof of Proposition 3.4 Following Axiom 3.3, money creation reduces the stock of foreign assets held by the central bank, and from Equation (16), a decrease in f_{cb} accelerates the total public debt ratio:

$$\frac{\partial \dot{b}}{\partial f_{cb}} = -\gamma.$$

□

Proof of Theorem 3.2 This follows logically from Equation (23). □

Proof of Proposition 3.5 This proof is straightforward and is shown below.

$$\overbrace{\frac{ln(2)}{\Big((i+g_q-\pi)-g\Big)}}^{\text{Pure Float: Augmented-Domar Condition}} < \overbrace{\frac{ln(2)}{(r-g)}}^{\text{Domar Condition}}$$

□

Proof of Proposition 3.6 This follows logically from Equation (24). Any primary balance that deviates from $\Omega^{f\,x\,r}_{ADC}$ is consistent with stock-flow disequilibria, where the public debt ratio and the nominal exchange rate are either exploding or decelerating, respectively. □

Proof of Proposition 3.7 Following Axiom 3.2, money creation increases the long-run rate of nominal depreciation, and accelerates the public debt ratio as it raises the local currency burden of external debt. This follows from Equation (23) as a nominal depreciation increases the weighted real interest rate. □